MACQUARIE
UNIVERSITY ~ SYDNEY

END-OF-YEAR EXAMINATIONS 2002

Unit: **ELEC321 Communication Systems (D2)**

Day and Time: Monday, 25 November 2002, 1:50 p.m.

Time Allowed: Three hours plus 10 minutes reading time.

Total Number of Questions: SIX (6)

Instructions: Answer any **FIVE** (5) questions only.

Total marks for this paper: 100.
The questions are of equal value.
Use one or more examination booklets.
Non-programmable electronic calculators may be used.

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1. Fourier Analysis

(a) (8 marks)

Find and sketch the autocorrelation function $R_{11}(\tau)$ for

$$x_1(t) = e^{-at}\, u(t),\ a > 0$$

where $u(t)$ is a unit step function.

(b) (8 marks)

Consider a filter with transfer function $H(f) = \frac{1}{1+j2\pi f}$ and an input $x(t) = e^{-2t}\, u(t)$, where $u(t)$ is a unit step function.

(i) Find the energy spectrum of the output.

(ii) Show that the normalised total energy output is one-third of the normalised input energy.

(c) (4 marks)

The transfer function of a particular n-th order low-pass filter is

$$|H(f)| = \frac{1}{\sqrt{1+(f/f_c)^{2n}}}$$

where f_c is the 3dB cutoff frequency.

(i) Show that, as $n \to \infty$, $|H(f)|$ approaches the characteristic of the ideal low-pass filter.

(ii) Find n so that $|H(f)|^2$ is constant to within 1 dB over the frequency range $|f| \leq 0.8\, f_c$.

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2. Analogue Modulation

(a) (10 marks)

Sketch block diagrams that show how you could produce the following signals; give a mathematical derivation of the operation of each system.

(i) A SSBSC signal using the phasing method;
(ii) A VSB signal.

(b) (10 marks)

Vestigial sideband (VSB) modulation is a compromise between DSB and SSB modulation. It relaxes the sharp-cutoff requirement of a SSB signal by retaining a trace of the other sideband in the transmitted signal. Let the Fourier transform of a VSB-modulated signal $s_c(t)$ be

$$S_c(f) = \frac{1}{2}[M(f-f_c) + M(f+f_c)]H(f)$$

where $M(f)$ is the Fourier transform of an input message signal $m(t)$, f_c is the carrier frequency, and $H(f)$ is the transfer function of a bandpass filter. Show that, for *distortionless* coherent demodulation of a VSB signal, $H(f)$ must satisfy the following condition:

$$H(f-f_c) + H(f+f_c) = K, \qquad |f| \leq B$$

where K is a constant and B is the bandwidth (in Hz) of the message signal.

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3. Nyquist Sampling Rate, PCM, and Delta Modulation

(a) (2 marks)

State the sampling theorem.

(b) (11 marks)

Find the Nyquist sampling rate for each of the following signals:

(i) $x_1(t) = 5 \cos 1000\pi t \cos 4000\pi t$;

(ii) $x_2(t) = \dfrac{\sin 200\pi t}{200\pi t}$;

(iii) $x_3(t) = \left(\dfrac{\sin 200\pi t}{200\pi t}\right)^2$.

(c) (4 marks)

A compact-disk (CD) recording system samples each of two stereo signals at 44100 samples per second. Each sample is then uniformly quantised into 16 bits.

(i) Determine the output peak signal-to-rms quantisation noise power ratio for a full-scale sinusoid.

(ii) Determine the number of bits recorded on a CD if the CD can record an hour's worth of music.

(d) (3 marks)

Consider a sinusoidal input test signal $x(t) = A \sin 2\pi f_m t$, where $A > 0$. If the sinusoidal signal is applied to a delta modulator with step size k', show that slope overload will occur if

$$k' f_s < 2\pi A f_m$$

where f_s is the sampling frequency.

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4. Line Coding and Waveform Shaping

(a) (2 marks)

A binary sequence may be encoded to various signal formats (called line codes) for transmission. Describe the difference between a return-to-zero (RZ) and a non-return-to-zero (NRZ) waveform format.

(b) (11 marks)

State the encoding rules for the following types of signals:

(i) Bipolar RZ signal;
(ii) Manchester-coded signal;
(iii) Miller-coded signal.

Figure 4.1 shows a Manchester-coded and a Miller-coded signal waveform, where T is the bit duration and $A > 0$. Determine the corresponding input binary sequences.

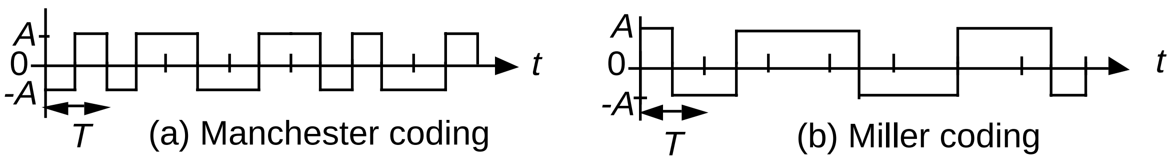

Figure 4.1 (a) Manchester coding and (b) Miller coding.

(c) (7 marks)

In digital communication, eye patterns (diagrams) are widely used as a qualitative/visual performance indicator of a system. Figure 4.2 shows an eye pattern for binary signals.

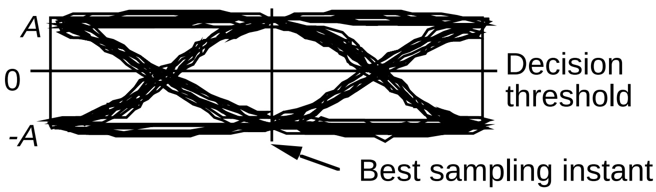

Figure 4.2 Eye pattern for binary signals.

Sketch the diagram in your answer and locate in it the eye opening, the noise margin of the system, the amount of amplitude distortion (ISI), and the amount of timing jitter.

Draw an eye pattern for 3-level signals.

5. Matched-Filter Detection

(a) (12 marks)

A known signal $s(t)$ together with additive white Gaussian noise with two-sided noise power spectral density $\frac{n_0}{2}$ watt/Hz is applied to the input of a matched filter. If the filter transfer function is $H(f)$ and its output is sampled at time t_0, use Parseval's Theorem and Schwarz's inequality to show that the maximum peak signal-to-noise power ratio at the output of the matched filter is

$$\frac{|s_o(t_0)|^2}{N} = \frac{2E}{n_0}$$

where E is the energy of the input signal $s(t)$, $|s_o(t_0)|$ is the amplitude of the output signal at time t_0, and N is the output noise power.

(b) (8 marks)

Find the output of the matched filter and find an expression for the maximum output signal-to-noise power ratio if the input $s(t)$ is a rectangular pulse of amplitude A and duration T.

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6. Source and Error-Control Coding

(a) (2 marks)

Write down the Huffman encoding procedure for a discrete memoryless source with symbols x_i and corresponding probabilities p_i, $i = 1, 2, ..., m$.

(b) (8 marks)

Consider a discrete memoryless source with symbols x_1, x_2, x_3, x_4, and x_5, and corresponding symbol probabilities $p_1 = 0.4$, $p_2 = 0.2$, $p_3 = 0.2$, $p_4 = 0.1$, and $p_5 = 0.1$. Construct a Huffman code for the source, and calculate the efficiency of the code.

(c) (10 marks)

Consider the binary block code C composed of the following four code words.

C = {(00100), (10010), (01001), (11111)}

(i) What is the number of information bits, k?

(ii) What is the number of parity-check bits, c ?

(iii) What is the minimum distance of this code?

(iv) What is the maximum weight for which the detection of all error patterns is guaranteed?

(v) What is the maximum weight for which the correction of all error patterns is guaranteed?

(vi) Is this code linear? Prove your answer.