MACQUARIE
UNIVERSITY ~ SYDNEY

END-OF-YEAR EXAMINATIONS 2003

Unit:	**ELEC321 Communication Systems (D2)**
Day and Time:	Monday, 17 November 2003, 1:50 p.m.
Time Allowed:	Three hours plus 10 minutes reading time.
Total Number of Questions:	SIX (6)
Instructions:	Answer any **FIVE** (5) questions only.
	Total marks for this paper: 100. The questions are of equal value. Use one or more examination booklets. Non-programmable electronic calculators may be used.

1. Fourier Series; Sampling

(a) (5 marks)

Working from first principles, determine the Fourier series for a periodic waveform consisting of rectangular pulses of height A and width t, repeated at intervals of T. Choose the time origin so that the waveform is an even function of time.

(b) (6 marks)

A signal $p(t)$ is given by the product of two signals $s_1(t)$ and $s_2(t)$:

$$p(t) = s_1(t) \times s_2(t) \quad .$$

Show that the spectrum $P(w)$ of $p(t)$ may be obtained by convolving the spectra $S_1(w)$ of $s_1(t)$ and $S_2(w)$ of $s_2(t)$.

(c) (5 marks)

A band-limited signal $f(t)$ is sampled at regular intervals of time T for a time interval t , so that for time intervals t the sampled signal $s(t)$ is equal to $f(t)$ but is zero otherwise.
Use your answers to parts (a) and (b) to describe the spectrum of $s(t)$.
Show how this result leads to the sampling theorem.

(d) (4 marks)

If the requirements of the sampling theorem are not satisfied, the phenomenon of ***aliasing*** may occur. Give two simple examples of aliasing.

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2. Analogue Modulation – AM and NFM

(a) (10 marks)

Consider a modulating signal $f(t) = A\cos\omega_m t$, where ω_m is the angular frequency of the modulating signal and A is a constant.

(i) Show that a narrow-band FM signal is given by

$$s(t) = \cos\omega_c t - \frac{\beta}{2}[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

where

ω_c is the angular frequency of the carrier signal,

$\beta = A/\omega_m$ is the modulation index

and

$\beta << \pi/2$.

State any assumptions you have to make.

(ii) Give a similar expression for an AM signal.

(b) (10 marks)

(i) Although AM and narrowband FM have similar frequency spectra, they are distinctly different modulation techniques. You are given a modulating signal

$f(t) = A\cos\omega_m t$

(where A may be varied) and a carrier signal

$s(t) = 1\cos\omega_c t$

to generate

- an AM signal with $m \times 100\%$ modulation

and

- a narrowband FM signal.

Using a vector representation, explain the difference between AM and narrowband FM signals.

(ii) The distinction and similarity between AM and narrowband FM leads to a common design method of generating AM and narrowband FM signals. Draw the block diagrams of an AM modulator and a narrowband FM modulator.

3. Pulse Amplitude Modulation; Pulse Code Modulation

(a) (8 marks)

Design a PAM multiplexing system for the transmission of three signals $m_1(t)$, $m_2(t)$ and $m_3(t)$, bandlimited to 8 kHz, 16 kHz and 8 kHz respectively. Assuming that each signal is sampled at twice its Nyquist sampling rate, find the commutator speed in rotations per second. Sketch a block diagram of the transmitter and receiver, and of the multiplexed signal waveform.

(b) (4 marks)

A piece of musical signal $m(t)$ is to be recorded by sampling at uniformly spaced instants of time and storing the sample values. Assuming that the highest-frequency component of $m(t)$ is 18500 Hz, what is the minimum number of samples that would be required to store five minutes of $m(t)$? If each sample is quantised into 256 levels, how many bits would be required to store the five minutes of $m(t)$?

(c) (2 marks)

Determine the mean-squared quantisation error of a PCM system with uniform quantisation. Assume that the error lies in the range $-a/2$ to $+a/2$, where a is the spacing between adjacent levels.

(d) (6 marks)

A differential-PCM transmission system uses a linear predictor; the predicted next value is obtained by extrapolating the straight line through the last two values. It uses a 4-bit sign-magnitude code, to cover values in the range $\pm 7 \times 0{\cdot}01$ V (i.e., integral multiples of 10 mV from –70 mV to +70 mV). The input $v(t)$ has been zero for a long time, but after $t = 0$ increases as shown below.

t (msec.)	0	1	2	3
$v(t)$ (V)	0·00	0·08	0·13	0·16

Explain how this system works, calculating the value of the 4-bit code sent by the transmitter at the three sampling instants ($t = 1, 2, 3$ msec.) in the above table.

4. Line Coding and Orthonormal Representation of Signals

(a) (8 marks)

Line coding involves converting (say) standard TTL/CMOS logic levels to a waveform more suitable for transmission. Describe the difference between a return-to-zero (RZ) and a non-return-to-zero (NRZ) waveform format. Given a binary sequence 1 1 0 0 0 1 0 1 1 1, describe and explain with diagrams what a polar NRZ signal, a polar NRZ-M signal and a Manchester-coded signal are.

(b) (12 marks)

Any set of m ***finite-energy*** signal waveforms $\{s_i(t), i = 1, 2, ..., m\}$ can be expressed as a weighted linear combination of orthonormal basis functions $\{u_j(t), j = 1, 2, ..., n\}$:

$$s_i(t) = \sum_{j=1}^{n} s_{ij}\, u_j(t)$$

where $m \leq n$. Figure 4 shows three signal waveforms $\{s_i(t), i = 1, 2, 3\}$.

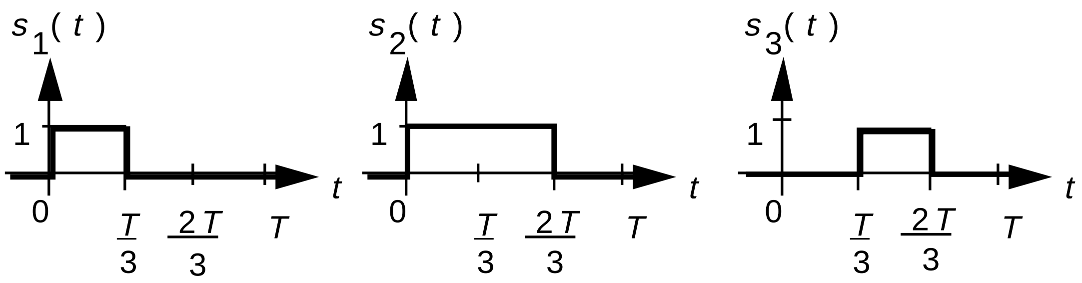

Figure 4 Signal set

Use the Gram-Schmitt Orthogonal process to compute the weighting coefficients $\{s_{ij}\}$ and the corresponding orthonormal basis functions $\{u_j(t)\}$. Also sketch all the orthonormal basis functions and draw the signal constellation diagram.

5. ASK Link Error Probability

(a) (12 marks)

An M-ary amplitude-shift-keying (M-ASK) signal symbol can be defined by

$$s_i(t) = \begin{cases} A_i \cos(2\pi f_c t), & 0 \le t \le T \\ 0, & elsewhere \end{cases}$$

where

$$A_i = A[2i - (M - 1)]$$

for $i = 0, 1, ..., M$-1. Here A is a constant, f_c is the carrier frequency, and T is the signalling interval.

The signal constellation for 4-ASK signal symbols is shown in Figure 5.

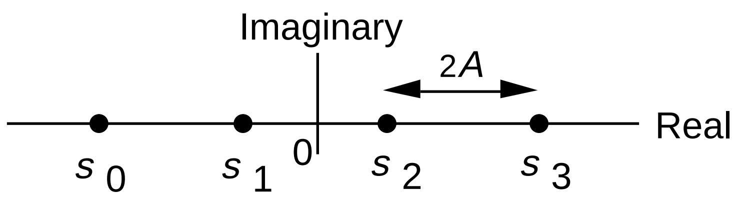

Figure 5 Signal constellation for 4-ASK signals

Use the signal-constellation diagram to derive an expression for the probability of symbol error in the presence of additive white Gaussian noise as a function of A and the mean square noise power, N. It is to be assumed that the signal symbols are equally likely to be transmitted.

(b) (8 marks)

A 4-ASK communication system has a bit rate of 20 Mbit/s. If A = 80 mV, and the noise power spectral density, N_0, is 32×10^{-12} W/Hz, calculate the symbol error probability. Assume that power and energy per symbol are normalised to a 1-Ω load.

[When calculating the symbol error probability, use the approximation $erfc(x) \approx \frac{e^{-x^2}}{x\sqrt{\pi}}$, for $x > 3$.]

6. Source and Error-Control Coding

(a) (2 marks)

Write down the Shannon-Fano encoding procedure for a discrete memoryless source with symbols x_i and corresponding probabilities p_i, $i = 1, 2, ..., m$.

(b) (6 marks)

Consider a discrete memoryless source with symbols

x_1, x_2, x_3, x_4 and x_5,

and corresponding symbol probabilities

$p_1 = 0.4$, $p_2 = 0.2$, $p_3 = 0.2$, $p_4 = 0.1$ and $p_5 = 0.1$.

If

x_1 encodes into 00,
x_2 encodes into 01,
x_3 encodes into 10,
x_4 encodes into 110, and
x_5 encodes into 111,

construct a Shannon-Fano encoding tree diagram, and calculate the efficiency of the code.

(c) (12 marks)

The parameters n and k of an (n, k) block code may be modified to suit a particular communication system. Name these modification processes and write a concise but informative account of these modification processes. You should include the following topics:

(i) How will the process affect the generator matrix of the code?
(ii) How, in some cases, will the process affect the minimum Hamming distance of the code?