MACQUARIE
UNIVERSITY ~ SYDNEY

END-OF-YEAR EXAMINATIONS 2006

Unit:	**ELEC321 Communication Systems (D2)**
Day and Time:	Tuesday, 28 November 2006, 9:20 a.m.
Time Allowed:	Three hours plus 10 minutes reading time.
Total Number of Questions:	SIX (6)
Instructions:	Answer any **FIVE** (5) questions only.
	Total marks for this paper: 100. The questions are of equal value. Use one or more examination booklets. Non-programmable electronic calculators may be used.

1. Fourier Analysis

(a) (5 marks)

Find the Fourier transform of the signum function which is defined as

$$\mathrm{sgn}(t) = \begin{cases} +1, & t > 0 \\ -1, & t < 0 \end{cases}$$

(b) (5 marks)

A unit step function is defined as

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

Verify that the Fourier transform of $u(t)$ is $\pi\delta(\omega) + \frac{1}{j\omega}$.

(c) (10 marks)

The Fourier transform of a signal $s(t)$ is given by

$$S(\omega) = \frac{1}{2}\Pi(\omega - \omega_0) + \frac{1}{2}\Pi(\omega + \omega_0)$$

where

$$\Pi(\omega) = \begin{cases} 1, & |\omega| < a \\ 0, & |\omega| > a \end{cases}$$

and $a < \omega_0$.

Find and sketch $s(t)$.

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2. Analogue Modulation - AM and NFM

(a) (10 marks)

Consider a modulating signal $f(t) = A \cos \omega_m t$, where ω_m is the angular frequency of the modulating signal and A is a constant.

(i) Show that a narrow-band FM signal is given by

$$s(t) \approx \cos \omega_c t - \frac{\beta}{2} [\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

where ω_c is the angular frequency of the carrier signal, $\beta = A\omega_m$ is the modulation index and $\beta << \pi/2$. State any assumptions you have to make.

(ii) Determine the bandwidth of the narrow-band FM signal.

(b) (10 marks)

(i) Although AM and narrow-band FM have similar frequency spectra, they are distinctly different modulation methods. You are given a modulating signal $f(t) = A \cos \omega_m t$ and a carrier signal $\cos \omega_c t$ to generate an AM signal with m x 100% modulation and a narrow-band FM signal. Using a vector representation, explain the difference between AM and narrow-band FM signals.

(ii) The distinction and similarity between AM and narrow-band FM leads to a common design method of generating AM and narrow-band FM signals. Draw the block diagrams of an AM modulator and a narrow-band FM modulator.

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3. Nyquist Sampling Rate and Pulse Code Modulation

(a) (12 marks)

The Nyquist sampling theorem states that the minimum sampling rate must be equal to or greater than twice the highest frequency component of a low-pass signal. When bandpass signals are to be sampled, a lower sampling rate can sometimes be used. Consider the bandpass signal $m(t)$ whose spectrum is shown in Figure 3.1. Sketch the spectrum of the *ideally* sampled signal $m_S(t)$ when the sampling frequency f_S is i) 25 kHz, ii) 45 kHz, and iii) 50 kHz. Indicate if and how the signal can be recovered in each case.

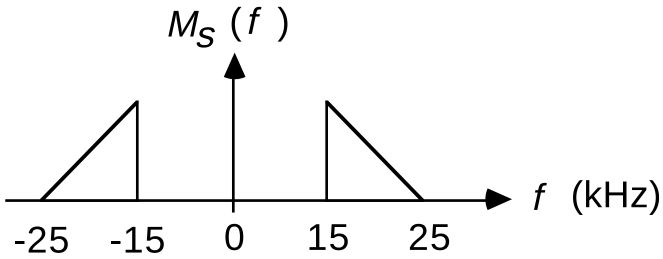

Figure 3.1 Spectrum of the bandpass signal $m(t)$

(b) (4 marks)

Explain why a non-uniform quantiser is used in a practical PCM system rather than a uniform quantiser.

(c) (4 marks)

Consider an analogue signal with frequency components limited to the range 300 Hz to 3300 Hz. The information in this signal is to be transmitted over a 16-ary PCM system. The quantisation distortion of the PCM system is specified to not exceed $p = 0.01$ times the peak-to-peak voltage of the analogue signal, so that

$$l \geq \log_2 \frac{1}{2p}$$

where l is the number of bits per sample.

(i) What is the minimum number of bits per sample that should be used in this PCM system?
(ii) What is the minimum required sampling rate?
(iii) What is the resulting transmission rate?
(iv) What is the PCM symbol transmission rate?

4. Line Coding and Waveform Shaping

(a) (3 marks)

Line coding involves converting standard TTL/CMOS logic levels to a suitable waveform for transmission. What primary factors should be considered when choosing or designing a line code?

(b) (11 marks)

Describe the difference between a return-to-zero (RZ) and a non-return-to-zero (NRZ) waveform format. Given a binary sequence 1 1 0 1 0 0 1, describe and explain with diagrams the following types of signals:

(i) Bipolar RZ signal;
(ii) Manchester coding;
(iii) Miller coding.

(c) (6 marks)

Suppose that the information transmission rate of a 16-point QAM signal employing a Nyquist shaping filter is 19.2 kbits/sec. Determine the transmission channel bandwidth if a sinusoidal roll-off factor of $r = 0.2$ is employed by the shaping filter.

5. PSK Link Error Probability

(a) (12 marks)

An M-ary phase-shift-keying (M-PSK) signal symbol can be defined by

$$s(t) = \begin{cases} A\cos\left(2\pi f_c t + \frac{2\pi}{M} i\right), & 0 \le t \le T \\ 0, & elsewhere \end{cases} \tag{5.1}$$

where i is one of 0, 1, ..., M-1. Here, A is a constant, f_c is the carrier frequency, and T is the signalling interval. The signal constellation for M-ary PSK signal symbols is shown in Figure 5.1.

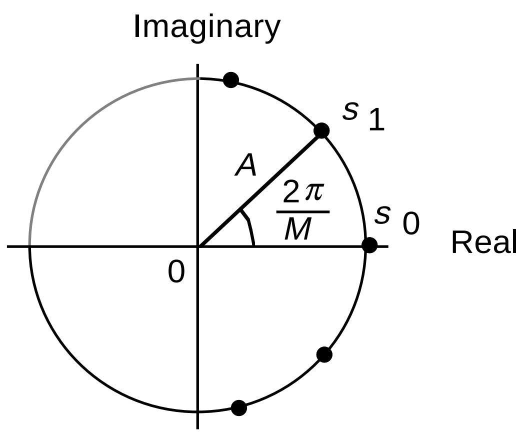

Figure 5.1 Signal constellation for M-ary PSK signals

Use the signal constellation diagram to derive an expression for the probability of symbol error in the presence of additive white Gaussian noise as a function of A and the mean square noise power, N. It is to be assumed that the signal symbols are equally likely to be transmitted.

(b) (8 marks)

An 8-PSK communication system has a bit rate of 20 Mbit/s. If A = 80 mV, and the noise power spectral density, N_0, is 2×10^{-12} W/Hz, calculate the symbol error probability.

Assume that power and energy per symbol are normalised to a 1-Ω load.

[When calculating the symbol error probability, use the approximation $erfc(x) \approx \frac{e^{-x^2}}{x\sqrt{\pi}}$, for $x > 3$.]

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6. Information Capacity, Source Coding and Error-Control Coding

(a) (2 marks)

A discrete memoryless source has four symbols x_1, x_2, x_3, x_4 with probabilities $p_1 = 0.4$, $p_2 = 0.3$, $p_3 = 0.2$, and $p_4 = 0.1$, respectively. Calculate the entropy of the source.

(b) (2 marks)

Write down the Shannon-Fano encoding procedure for a discrete memoryless source with symbols x_i and corresponding probabilities p_i, $i = 1, 2, ..., m$.

(c) (4 marks)

Consider a discrete memoryless source with symbols x_1, x_2, x_3, and x_4, and corresponding symbol probabilities $p_1 = 0.125$, $p_2 = 0.5$, $p_3 = 0.25$, and $p_4 = 0.125$. Construct a Shannon-Fano code for the source.

(d) (12 marks)

A binary single-error-correcting Hamming code has block length $n = 15$. What is

(i) the number of parity-check digits, c ?
(ii) the number of information digits, k ?
(iii) the parity-check matrix, $\mathbf{H}$?
(iv) the binary representation of the syndrome if the fourth information digit is in error ?
