

Some Useful Formulas and Functions

Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j \sin x$$

$$\cos x = \sin(x + 90^\circ) = \frac{1}{2} (e^{jx} + e^{-jx})$$

$$\sin x = \cos(x - 90^\circ) = \frac{1}{2j} (e^{jx} - e^{-jx})$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \frac{1}{2} (1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2} (1 - \cos 2x)$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$2 \cos x \cos y = \cos(x - y) + \cos(x + y)$$

$$2 \sin x \sin y = \cos(x - y) - \cos(x + y)$$

$$2 \sin x \cos y = \sin(x - y) + \sin(x + y)$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Series Expansions

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \dots, |nx| < 1$$

$$e^x = 1 + x + \frac{1}{2!} x^2 + \dots$$

$$a^x = 1 + x \ln a + \frac{1}{2!} (x \ln a)^2 + \dots$$

$$\ln(1 + x) = x - \frac{1}{2} x^2 + \frac{1}{3} x^3 - \dots$$

$$\cos x = 1 - \frac{1}{2!} x^2 + \frac{1}{4!} x^4 - \dots$$

$$\sin x = x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 - \dots$$

$$J_n(x) = \frac{1}{n!} \left(\frac{x}{2}\right)^n - \frac{1}{(n+1)!} \left(\frac{x}{2}\right)^{n+2} + \frac{1}{2!(n+2)!} \left(\frac{x}{2}\right)^{n+4} - \dots$$

Error and Complementary Error Functions

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\alpha^2} d\alpha, \quad \operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-\alpha^2} d\alpha$$

x	$\operatorname{erf}(x)$	$\operatorname{erfc}(x)$	x	$\operatorname{erf}(x)$	$\operatorname{erfc}(x)$
0.05	0.05637	0.944	1.55	0.97162	0.0284
0.10	0.11246	0.888	1.60	0.97635	0.0238
0.15	0.16800	0.832	1.65	0.98038	0.0196
0.20	0.22270	0.777	1.70	0.98379	0.0156
0.25	0.27633	0.724	1.75	0.98667	0.0133
0.30	0.32863	0.671	1.80	0.98909	0.0105
0.35	0.37938	0.621	1.85	0.99111	8.89×10^{-3}
0.40	0.42839	0.572	1.90	0.99279	6.91×10^{-3}
0.45	0.47548	0.525	1.95	0.99418	5.82×10^{-3}
0.50	0.52050	0.479	2.00	0.99532	4.59×10^{-3}
0.55	0.56332	0.437	2.05	0.99626	3.74×10^{-3}
0.60	0.60386	0.396	2.10	0.99702	2.93×10^{-3}
0.65	0.64203	0.358	2.15	0.99764	2.36×10^{-3}
0.70	0.67780	0.322	2.20	0.99814	1.84×10^{-3}
0.75	0.71116	0.289	2.25	0.99854	1.46×10^{-3}
0.80	0.74210	0.258	2.30	0.99886	1.13×10^{-3}
0.85	0.77067	0.230	2.35	0.99911	8.9×10^{-4}
0.90	0.79691	0.203	2.40	0.99931	6.82×10^{-4}
0.95	0.82089	0.180	2.45	0.99947	5.3×10^{-4}
1.00	0.84270	0.157	2.50	0.99959	4.03×10^{-4}
1.05	0.86244	0.138	2.55	0.99968	3.11×10^{-4}
1.10	0.88021	0.120	2.60	0.99976	2.34×10^{-4}
1.15	0.89612	0.104	2.65	0.99982	1.78×10^{-4}
1.20	0.91031	0.0901	2.70	0.99987	1.33×10^{-4}
1.25	0.92290	0.0771	2.75	0.99990	1.01×10^{-4}
1.30	0.93401	0.0659	2.80	0.99993	7.5×10^{-5}
1.35	0.94376	0.0562	2.85	0.99994	5.57×10^{-5}
1.40	0.95229	0.0481	2.90	0.99996	4.09×10^{-5}
1.45	0.95970	0.0403	2.95	0.99997	3.02×10^{-5}
1.50	0.96611	0.0338	3.00	0.99998	2.21×10^{-5}

Bessel Functions

β	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}
0.0	1.00										
0.2	0.99	0.10									
0.4	0.96	0.20	0.02								
0.6	0.91	0.29	0.04								
0.8	0.85	0.37	0.08	0.01							
1.0	0.77	0.44	0.11	0.02							
1.2	0.67	0.50	0.16	0.03	0.01						
1.4	0.57	0.54	0.21	0.05	0.01						
1.6	0.46	0.57	0.26	0.07	0.01						
1.8	0.34	0.58	0.31	0.1	0.02						
2.0	0.22	0.58	0.35	0.13	0.03	0.01					
2.2	0.11	0.56	0.40	0.16	0.05	0.01					
2.4	0.00	0.52	0.43	0.20	0.06	0.02					
2.6	-0.10	0.47	0.46	0.24	0.08	0.02	0.01				
2.8	-0.19	0.41	0.48	0.27	0.11	0.03	0.01				
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01				
3.2	-0.32	0.26	0.48	0.34	0.16	0.06	0.02				
3.4	-0.36	0.18	0.47	0.37	0.19	0.07	0.02	0.01			
3.6	-0.39	0.10	0.44	0.40	0.22	0.09	0.03	0.01			
3.8	-0.40	0.01	0.41	0.42	0.25	0.11	0.04	0.01			
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02			
4.2	-0.38	-0.14	0.31	0.43	0.31	0.16	0.06	0.02	0.01		
4.4	-0.34	-0.20	0.25	0.43	0.34	0.18	0.08	0.03	0.01		
4.6	-0.30	-0.26	0.18	0.42	0.36	0.21	0.09	0.03	0.01		
4.8	-0.24	-0.30	0.12	0.40	0.38	0.23	0.11	0.04	0.01		
5.0	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	0.01	
5.2	-0.11	-0.34	-0.02	0.33	0.40	0.29	0.15	0.07	0.02	0.01	
5.4	-0.04	-0.35	-0.09	0.28	0.40	0.31	0.18	0.08	0.03	0.01	
5.6	0.03	-0.33	-0.15	0.23	0.39	0.33	0.20	0.09	0.04	0.01	
5.8	0.09	-0.31	-0.20	0.17	0.38	0.35	0.22	0.11	0.05	0.02	0.01
6.0	0.15	-0.28	-0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	0.01
6.2	0.20	-0.23	-0.28	0.05	0.33	0.37	0.27	0.15	0.07	0.03	0.01
6.4	0.24	-0.18	-0.30	-0.01	0.29	0.37	0.29	0.17	0.08	0.03	0.01
6.6	0.27	-0.12	-0.31	-0.06	0.25	0.37	0.31	0.19	0.10	0.04	0.01
6.8	0.29	-0.07	-0.31	-0.12	0.21	0.36	0.33	0.21	0.11	0.05	0.02

Bessel Functions (Continued)

β	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}
7.0	0.30	-0.00	-0.30	-0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02
7.2	0.30	0.05	-0.28	-0.21	0.11	0.33	0.35	0.25	0.15	0.07	0.03
7.4	0.28	0.11	-0.25	-0.24	0.05	0.30	0.35	0.27	0.16	0.08	0.04
7.6	0.25	0.16	-0.21	-0.27	-0.00	0.27	0.35	0.29	0.18	0.10	0.04
7.8	0.22	0.20	-0.16	-0.29	-0.06	0.23	0.35	0.31	0.20	0.11	0.05
8.0	0.17	0.23	-0.11	-0.29	-0.11	0.19	0.34	0.32	0.22	0.13	0.06
8.2	0.12	0.26	-0.06	-0.29	-0.15	0.14	0.32	0.33	0.24	0.14	0.07
8.4	0.07	0.27	-0.00	-0.27	-0.19	0.09	0.30	0.34	0.26	0.16	0.08
8.6	0.01	0.27	0.05	-0.25	-0.22	0.04	0.27	0.34	0.28	0.18	0.10
8.8	-0.04	0.26	0.10	-0.22	-0.25	-0.01	0.24	0.34	0.29	0.20	0.11
9.0	-0.09	0.25	0.14	-0.18	-0.27	-0.06	0.20	0.33	0.31	0.21	0.12
9.2	-0.14	0.22	0.18	-0.14	-0.27	-0.10	0.16	0.31	0.31	0.23	0.14
9.4	-0.18	0.18	0.22	-0.09	-0.27	-0.14	0.12	0.30	0.32	0.25	0.16
9.6	-0.21	0.14	0.24	-0.04	-0.26	-0.18	0.08	0.27	0.32	0.27	0.17
9.8	-0.23	0.09	0.25	0.01	-0.25	-0.21	0.03	0.25	0.32	0.28	0.19
10.0	-0.25	0.04	0.25	0.06	-0.22	-0.23	-0.01	0.22	0.32	0.29	0.21

Reference

- [1] J. D. Gibson, Principles of Digital and Analog Communications, 2/e, Macmillan Pubs., 1993, ISBN 0-02-341860-5.

Hilbert Transform

$$\hat{m}(t) = H[m(t)] = m(t) * \frac{1}{\pi t} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{m(\tau)}{t - \tau} d\tau$$

Function	Hilbert transform
$m(at + b)$	$\hat{m}(at + b)$
$m_1(t) + m_2(t)$	$\hat{m}_1(t) + \hat{m}_2(t)$
$\frac{d^n m(t)}{dt^n}$	$\frac{d^n \hat{m}(t)}{dt^n}$
a	0
$\frac{1}{t}$	$-\pi\delta(t)$
$\sin(2\pi f_c t + \phi)$	$-\cos(2\pi f_c t + \phi)$
$\text{Sa}(t) = \frac{\sin at}{at}$	$-\frac{1}{2\pi} \text{Sa}^2(at)$
$e^{\pm j2\pi f_c t}$	$\mp j e^{\pm j2\pi f_c t}$
$\delta(t)$	$\frac{1}{\pi t}$
$\frac{a}{\pi(t^2 + a^2)}$	$\frac{t}{\pi(t^2 + a^2)}$
$\Pi\left(\frac{t}{T}\right)$	$\frac{1}{\pi} \ln \left \frac{2 - \frac{t}{T}}{2 + \frac{t}{T}} \right $

Reference

- [1] L. W. Couch II, Digital and Analog Communication Systems, 5/e, Prentice Hall, 1997.