6. Amplitude Modulation

Modulation is a process by which some parameter of a *carrier signal* is varied in accordance with a message signal. The message signal is called a *modulating signal*.

Definitions

A bandpass signal is represented by

$$s_c(t) = A(t)\cos\theta(t) \tag{6.1}$$

where $A(t)$ is the envelope and $\theta(t) = \omega_c t + \phi(t) = 2\pi f_c t + \phi(t)$. $\phi(t)$ is called the *instantaneous phase deviation* of $s_c(t)$ and f_c is the carrier frequency. For amplitude modulation, we can write

$$s_c(t) = A(t)\cos 2\pi f_c t \tag{6.2}$$

where $A(t)$ is linearly related to the modulating signal $m(t)$. $A(t)$ is called the *instantaneous amplitude* of $s_c(t)$ and amplitude modulation is also referred to as *linear modulation*. Depending on the relationship between $m(t)$ and $A(t)$, we have the following types of amplitude modulation schemes: *normal amplitude modulation* (AM), *double-sideband* (DSB) modulation, *single-sideband* (SSB) modulation, and *vestigial-sideband* (VSB) modulation.

Normal Amplitude Modulation (AM)

A normal amplitude-modulated signal is given by

$$s_c(t) = [A + m(t)]\cos 2\pi f_c t \tag{6.3}$$

$$s_c(t) = \underbrace{A\cos 2\pi f_c t}_{carrier} + \underbrace{m(t)\cos 2\pi f_c t}_{sidebands} \tag{6.4}$$

where A is a constant and $m(t)$ is the modulating signal. It is also very common to define a normal amplitude-modulated signal as

$$s_c(t) = A[1 + m(t)]\cos 2\pi f_c t \tag{6.5}$$

The *modulation index* m is defined as [1]

$$m = \frac{|\min m(t)|}{A} \tag{6.6}$$

Figure 6.1 shows normal AM signals for various values of modulation index. Clearly, the envelope of the modulated signals has the same shape as $m(t)$ when $m \leq 1$. When $m > 1$, the carrier signal is said to be *overmodulated* and the envelope is distorted.

Figure 6.1 Normal AM signals for various values of modulation index.

The efficiency η of a normal AM signal is defined as [2]

$$\eta = \frac{P_s}{P_t} \text{ x } 100\% \qquad (6.7)$$

where P_s is the power carried by the sidebands and P_t is the total power of the normal AM signal.

Spectrum of Normal AM Signals

For normal amplitude modulation,

$$s_c(t) = [A + m(t)] \cos 2\pi f_c t \qquad (6.8)$$

$$s_c(t) = A \cos 2\pi f_c t + m(t) \cos 2\pi f_c t$$

The Fourier transform of $s_c(t)$ is

$$S_c(f) = \frac{1}{2} A[\delta(f-f_c) + \delta(f+f_c)] + \frac{1}{2} [M(f-f_c) + M(f+f_c)] \qquad (6.9)$$

Figure 6.2 shows the spectrum of a normal AM signal. Normal amplitude modulation simply shifts the spectrum of $m(t)$ to the carrier frequency f_c. The bandwidth of the modulated signal is 2 f_m Hz, where f_m is the bandwidth of the modulating signal $m(t)$.

Figure 6.2 Spectrum of normal AM signal.

Generation of Normal AM Signals

A process of generating a normal AM signal is shown in Figure 6.3. This type of modulation can be achieved by using a non-linear device, such as a diode. This is shown in Figure 6.4 [3].

Figure 6.3 Generation of normal AM signal.

Figure 6.4 Amplitude modulator using a diode.

Let the input-output characteristic of a diode be approximated by a power series

$$v_o(t) = a\, v_i(t) + b\, v_i^2(t) \tag{6.10}$$

where a, b are constants and

$$v_i(t) = \cos 2\pi f_c t + m(t) \tag{6.11}$$

Substituting equation (6.11) into (6.10), we have

$$\begin{aligned} v_o(t) = &\ a[\cos 2\pi f_c t + m(t)] + b[\cos 2\pi f_c t + m(t)]^2 \\ = &\ a\, m(t) + b\cos^2 2\pi f_c t + b\, m(t)^2 + \\ &\ a\cos 2\pi f_c t + 2b\, m(t) \cos 2\pi f_c t \end{aligned} \tag{6.12}$$

If we pass the signal $v_o(t)$ through a bandpass filter centred at $\pm f_c$, we obtain

$$v'_o(t) = [a + 2b\, m(t)] \cos 2\pi f_c t \tag{6.13}$$

$$= 2b[A + m(t)] \cos 2\pi f_c t \tag{6.14}$$

where $A = \frac{a}{2b}$. We generate a normal AM signal.

A normal amplitude-modulated signal can also be obtained by multiplying $m(t)$ by a periodic digital signal $s(t)$. The modulator is called a *switching modulator* [2]. If we take a periodic rectangular waveform $s(t)$ of period $T_c = 1/f_c$, amplitude A_m, and pulse width τ, the trigonometric Fourier series of $s(t)$ is

$$s(t) = \frac{A_m\tau}{T_c} + \frac{2}{T_c}\sum_{n=1}^{\infty}\left(A_m\tau\frac{\sin 2\pi n f_c\tau/2}{2\pi n f_c\tau/2}\right)\cos 2\pi n f_c t \tag{6.15}$$

$$s(t) = \frac{c_0}{T_c} + \frac{2}{T_c}\sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t \tag{6.16}$$

where $c_0 = A_m\tau$ and $c_n = A_m\tau\frac{\sin 2\pi n f_c\tau/2}{2\pi n f_c\tau/2}$. The corresponding complex Fourier series is

$$s(t) = \frac{1}{T_c}\sum_{n=-\infty}^{\infty}\left(A_m\tau\frac{\sin 2\pi n f_c\tau/2}{2\pi n f_c\tau/2}\right)e^{j2\pi n f_c t} \tag{6.17}$$

Figure 6.5 shows the periodic rectangular waveform and its line spectrum.

Figure 6.5 (a) A periodic rectangular waveform, and (b) its line spectrum.

If the input signal is $v_i(t) = \cos 2\pi f_c t + m(t)$, the output of a switching modulator is

$$\begin{aligned}
v_o(t) &= v_i(t)\, s(t) \\
&= [\cos 2\pi f_c t + m(t)]\, s(t) \\
&= [\cos 2\pi f_c t + m(t)]\left(\frac{c_0}{T_c} + \frac{2}{T_c}\sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t\right) \\
&= \left[\frac{2}{T_c}\cos 2\pi f_c t \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t\right] + \frac{c_0}{T_c}\, m(t) + \\
&\quad \frac{c_0}{T_c}\cos 2\pi f_c t + m(t)\frac{2}{T_c}\sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t
\end{aligned}$$

$v_o(t)$ consists of a dc term, the component $m(t)$, and an infinite number of normal AM signals at carrier frequencies f_c, $2f_c$, $3f_c$, ... If we pass the signal $v_o(t)$ through a bandpass filter centred at $\pm f_c$, the filtered signal is

$$\begin{aligned}
v'_o(t) &= \frac{c_0}{T_c}\cos 2\pi f_c t + \frac{2}{T_c}\, m(t) c_1 \cos 2\pi f_c t + \frac{c_2}{T_c}\cos 2\pi f_c t \\
&= \frac{2c_1}{T_c}[A + m(t)] \cos 2\pi f_c t
\end{aligned}$$

where $A = \frac{c_0 + c_2}{2c_1}$. That is, we obtain a normal AM signal.

Demodulation of Normal AM Signals [3]

The process of recovering the message signal from the modulated signal is called *demodulation* or *detection*. Two basic methods are available.

Envelope Detection.

In this method, an envelope detector is used to recover the message signal. An envelope detector consists of a diode and a resistor-capacitor combination. This is shown in Figure 6.6.

Figure 6.6 Envelope detector.

During the positive half-cycle peaks of the modulated signal, the diode is forward biased, and the capacitor charges up to the peak value of the modulated signal. As the modulated signal falls from its maximum, the diode turns off and the capacitor discharges through the resistor. The process repeats in this way. For proper operation, the discharge time constant *RC* must be chosen properly.

Synchronous (Coherent) Detection.

Here, a product detector is used to convert the bandpass signal to baseband. This is shown in Figure 6.7.

Figure 6.7 Synchronous detector.

At the receiving end, the bandpass signal is multiplied by a locally generated carrier signal $\cos(2\pi f_c t + \phi_0)$, where ϕ_0 is an initial phase. The output of the multiplier is

$$\begin{aligned} x(t) &= [A + m(t)] \cos 2\pi f_c t \cos(2\pi f_c t + \phi_0) \\ &= 0.5[A + m(t)] [\cos \phi_0 + \cos(4\pi f_c t + \phi_0)] \\ &= 0.5[A + m(t)] \cos(4\pi f_c t + \phi_0) + 0.5A\cos \phi_0 + 0.5m(t)\cos \phi_0 \end{aligned} \tag{6.18}$$

If we suppress the first term by a low-pass filter, we get

$$y(t) = 0.5A\cos \phi_0 + 0.5m(t)\cos \phi_0 \tag{6.19}$$

It can be seen that we can recover the component $m(t)$ if the initial phase ϕ_0 is constant and small. Suppose that the local carrier signal is $\cos[2\pi(fc + \Delta f)t]$, when the multiplier output becomes

$$\begin{aligned} x(t) &= [A + m(t)] \cos 2\pi f_c t \cos[2\pi(fc + \Delta f)t] \\ &= 0.5[A + m(t)] [\cos 2\pi\Delta f\, t + \cos 2\pi(2f_c + \Delta f)t] \\ &= 0.5[A + m(t)] \cos 2\pi(2f_c + \Delta f)t + \\ &\quad 0.5A\cos 2\pi\Delta f\, t + 0.5m(t)\cos 2\pi\Delta f\, t \end{aligned} \tag{6.20}$$

If we suppress the first term by a low-pass filter, we get

$$y(t) = 0.5A\cos 2\pi\Delta f\, t + 0.5m(t)\cos 2\pi\Delta f\, t \tag{6.21}$$

We cannot recover the component $m(t)$ unless the frequency drift Δf is zero. Therefore, the local carrier must not only be of the same frequency but must be synchronised in phase with the carrier signal. If the carrier shifts in frequency or phase, the resultant signal is distorted or attenuated. Synchronous detection is sometimes called *coherent detection*.

References

[1] H. P. Hsu, Analog and Digital Communications, McGraw-Hill, 1993.

[2] L. W. Couch II, Digital and Analog Communication Systems, 5/e, Prentice Hall, 1997.

[3] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw-Hill, 1990.

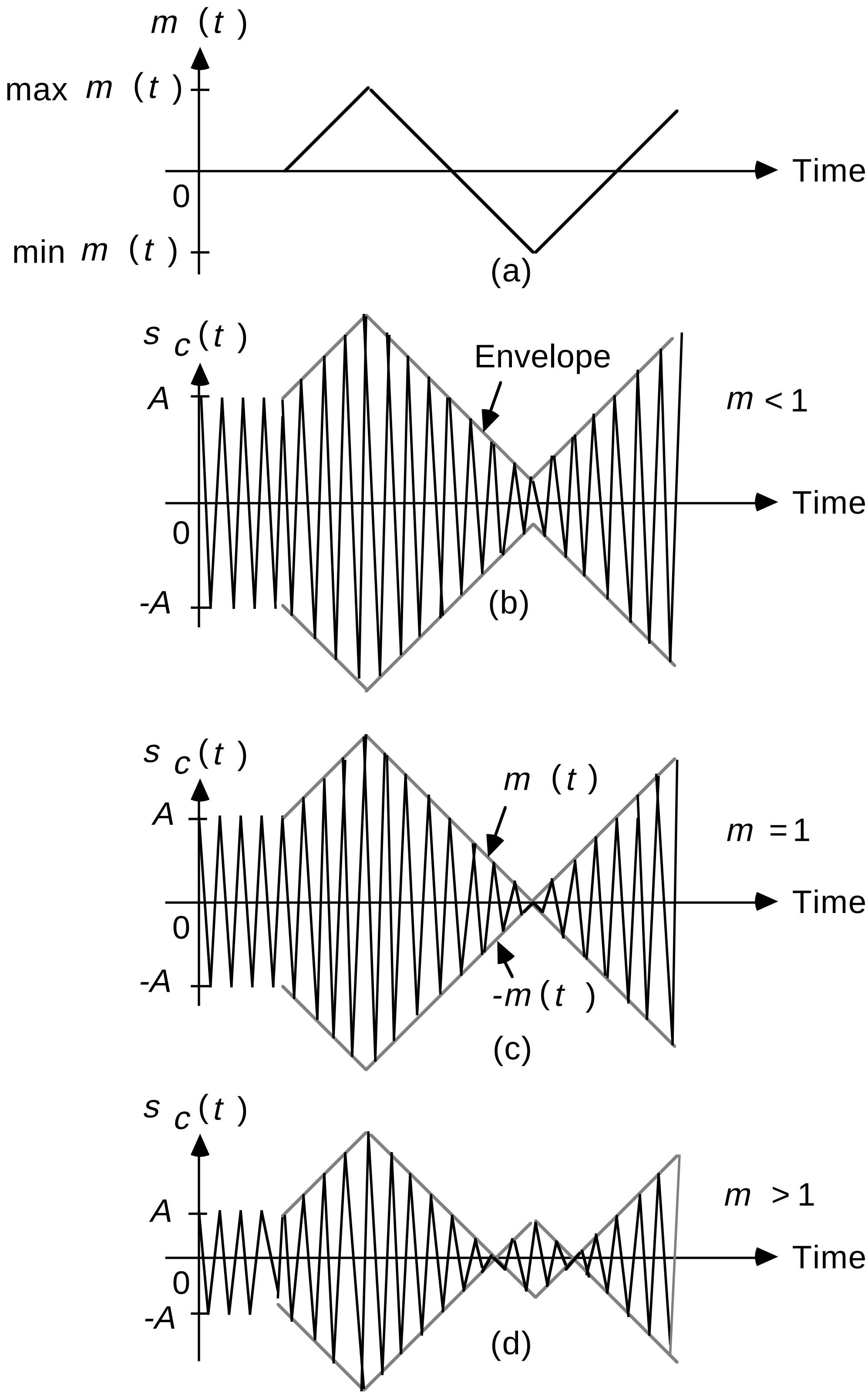

Figure 6.1 Normal AM signals for various values of modulation index.

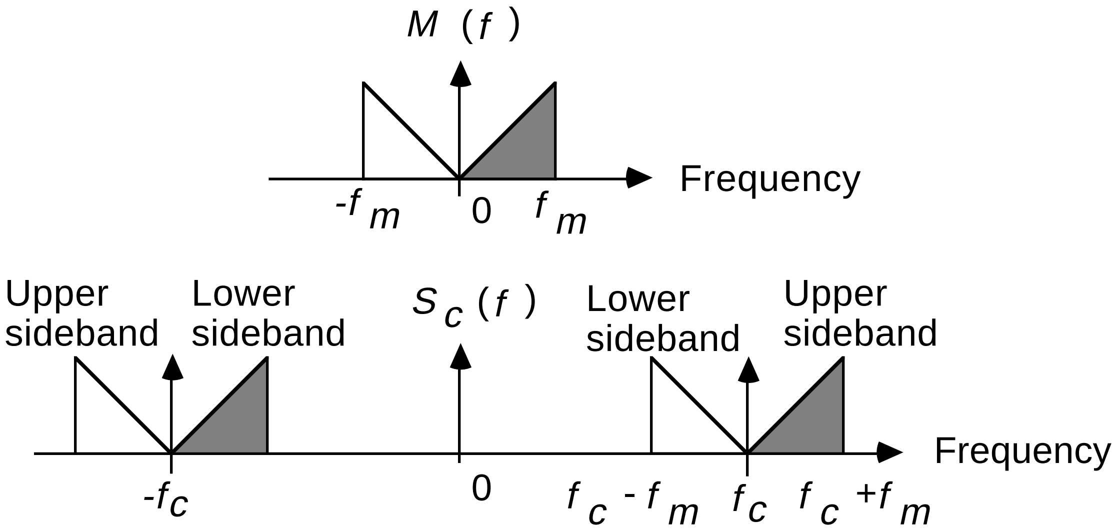

Figure 6.2 Spectrum of normal AM signal.

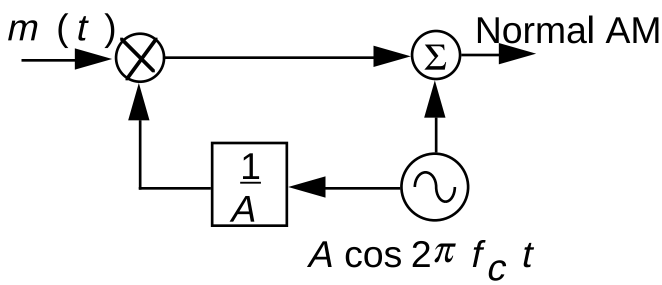

Figure 6.3 Generation of normal AM signal.

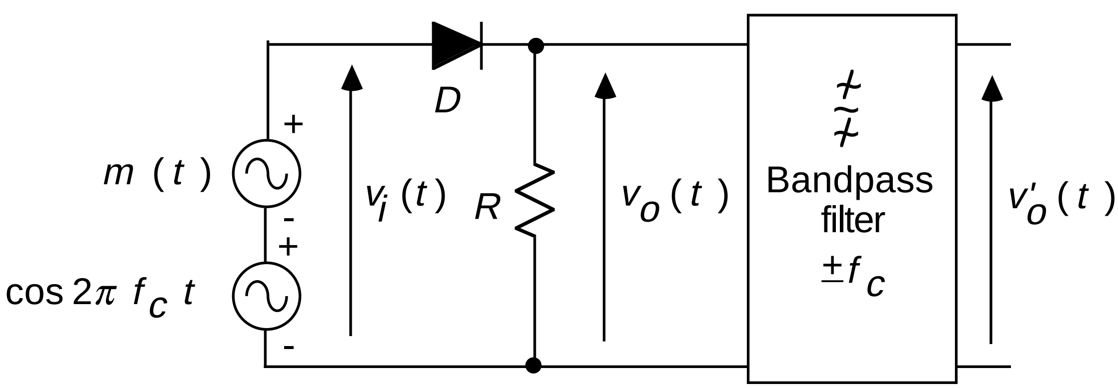

Figure 6.4 Amplitude modulator using a diode.

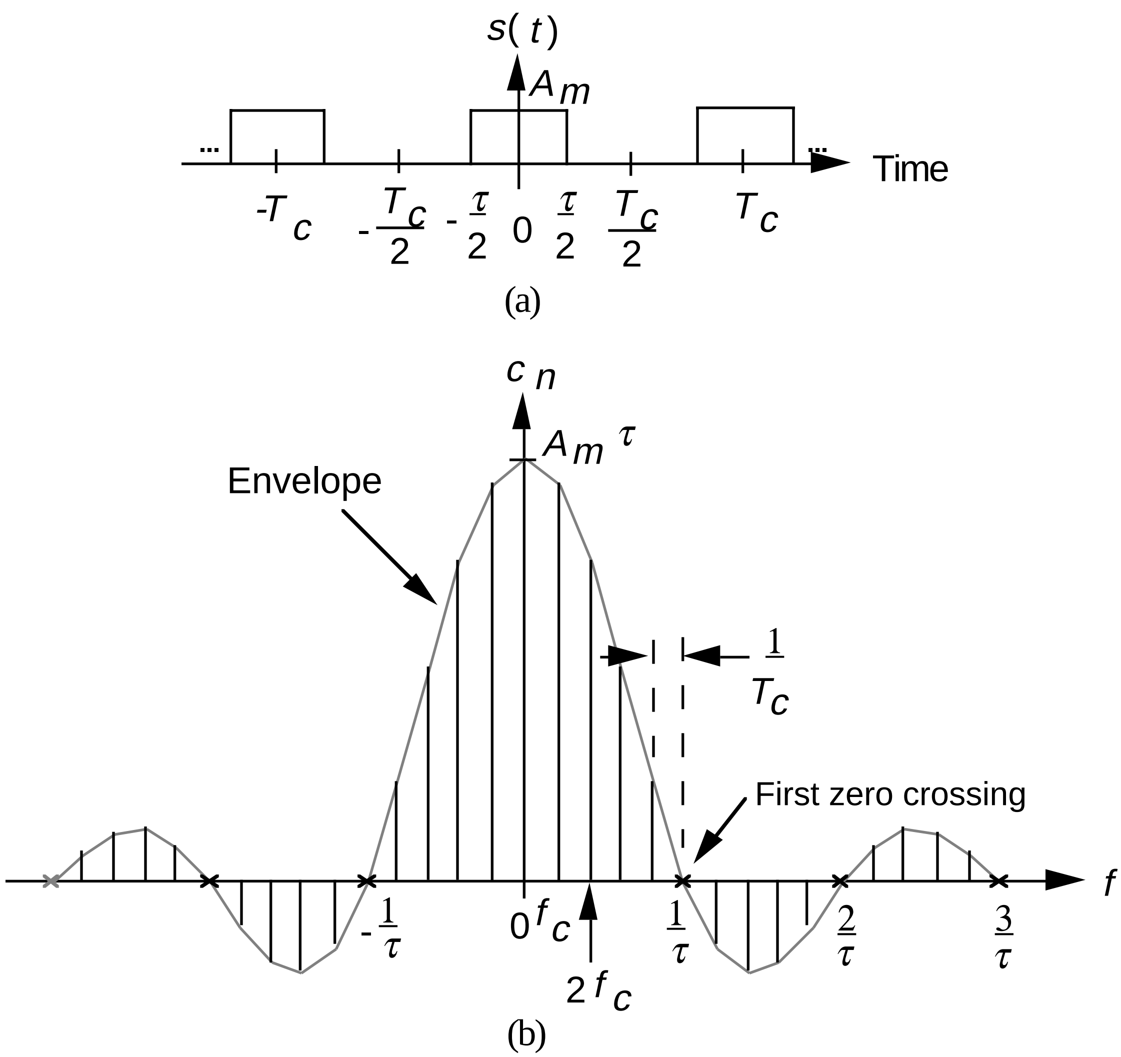

Figure 6.5 (a) A periodic rectangular waveform, and (b) its line spectrum.

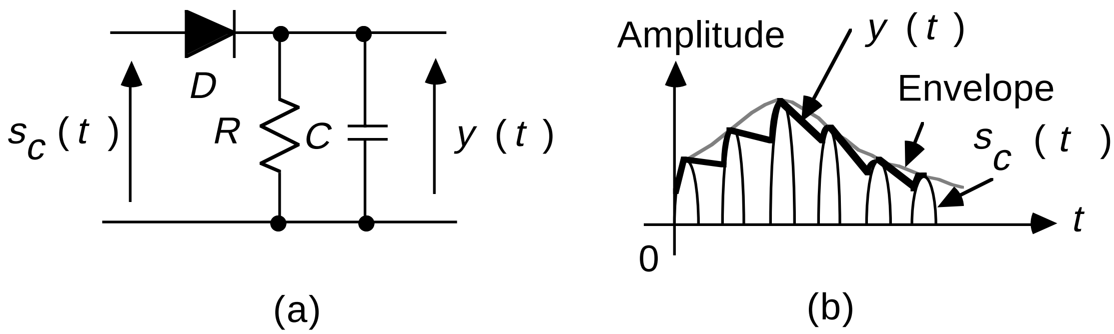

Figure 6.6 Envelope detector.

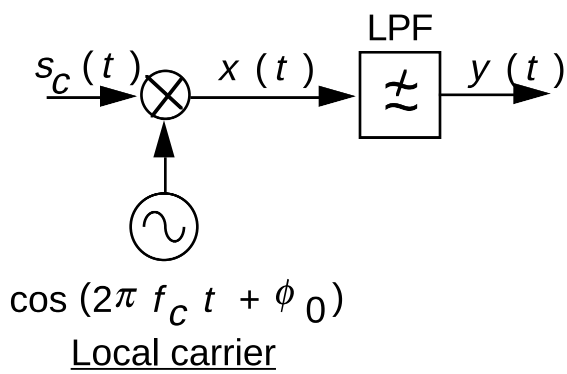

Figure 6.7 Synchronous detector.