17. Delta Modulation

Introduction

So far, we have seen that the pulse-code-modulation (PCM) technique converts analogue signals to digital format for transmission. For speech signals of 3.2kHz bandwidth, we usually sample at 8000 sample/s and employ an 8-bit quantiser. The overall transmission rate is 64 kbit/s. To reduce the bandwidth and the cost, and to improve system performance, *delta modulation* (DM) and *differential pulse code modulation* (DPCM) have been used [1-3].

Delta Modulation

Consider the delta modulator and demodulator shown in Figure 17.1. For simplicity, the coefficients $h_2, h_3, ..., h_K$ of the predictor are set to 0 and the associated signal waveforms of the delta modulator are shown in Figure 17.2, where the *predictor* output sample g_j is available at time sampling instant j. Let x_j and $\hat{x}_j$ be the sampled signal and the sample estimate of x_j at time j, respectively.

Figure 17.1 (a) Delta modulator. (b) Delta demodulator.

Figure 17.2 Operation of delta modulator.

An analogue signal $x(t)$ is first sampled periodically. Each sampled value x_j is compared with a predicted sample value g_j and the difference ε_j is then passed to a quantiser. If the predicted value g_j is close to the sampled value x_j, ε_j is small and fewer bits will be needed to represent ε_j. In delta modulation, a 2-level quantiser is used to represent ε_j. When ε_j is positive, the output of the quantiser is $+k'$. When ε_j is negative, the output of the quantiser is $-k'$. At the receiving end, the quantised signal is added to the predictor output to obtain a discrete estimate $\hat{x}_j$ of the desired sampled signal x_j. A low-pass filter is then used to recover $x(t)$.

In delta modulation, the predictor simply weights the sum of past sample estimates. In its general form,

$$g_j = \sum_{l=1}^{K} h_l \hat{x}_{j-l} \tag{17.1}$$

where $\{h_l\}$ are the weighting factors and $\{\hat{x}_{j-l}\}$ are the past sample estimates for $1 \le l \le K$. The weighting factors are chosen to reduce some measure of the estimate errors.

One common measure is the mean-squared error.

To improve the system performance, we can use a multi-level quantiser. This is called differential PCM (DPCM) [2-3]. Delta modulation is therefore a class of DPCM.

DM and DPCM have been applied to digital image transmission. In these applications, there is a great deal of redundancy in the information to be sent. Past information can be used to predict current information. For transmission of high-quality speech, the International Telegraph and Telephone Consultative Committee (CCITT) recommends 32 kbit/s adaptive DPCM with 4-bit quantisation for 3.2kHz speech and 64 kbit/s adaptive DPCM with 4-bit quantisation for 7kHz speech.

When high-quality speech is not a prime factor, delta modulation is recommended. We shall say no more on DPCM and focus on DM.

Types of Noise

1. In delta modulation, *quantisation noise* similar to PCM quantisation noise appears at the receiver output. This is shown in Figure 17.3(a). We can reduce the quantisation noise by increasing the number of quantisation levels in a PCM system. In delta modulation, the quantiser is fixed at 2 levels. We can reduce the quantisation noise in a delta modulation system by reducing the *step size* k' or by increasing the *sampling rate*. In practice, delta modulation uses a higher sampling rate than pulse-code modulation.

   Another technique to reduce the quantisation noise in a delta modulation system is to employ an adaptive delta modulator [2, 3]. In this scheme, the quantisation step is varied according to the output of the quantiser.

Figure 17.3 (a) Quantisation noise. (b) Overload noise.

2. The second type of noise in a delta modulation system is called *overload noise*. This is shown in Figure 17.3(b). If the quantisation levels $\pm k'$ are too small to track a rapidly changing signal, the estimated value $\hat{x}_j$ will not follow x_j.

   The overload occurs because the step size k' sets an upper limit on the slope of the input signal $x(t)$ that the modulator can follow. If the sampling rate is f_s samples/s, the time between samples is $1/f_s$ seconds. The slope is

$$k' / (1/f_s) = k' f_s \qquad (17.2)$$

It can be seen in Figure 17.3 that increasing the step size k' increases the quantisation noise but reduces overload noise. On the other hand, decreasing the step size k' decreases the quantisation noise at the expense of introducing overload noise. This gives rise to an optimum choice of k'. A typical performance curve of a delta modulation system is shown in Figure 17.4.

Figure 17.4 Typical performance curve for delta modulation.

In practice, one of the following procedures can be used to minimise the quantisation and overload noise.

1. We fix the step size k' and use a higher sampling rate. This reduces quantisation noise without introducing overload noise.

2. We employ an adaptive delta modulator to vary the step size. In adaptive delta modulation, the step size k' is kept small until slope overload begins, corresponding to a string of positive quantisation output values. The step size is then increased. If the output of the quantiser is alternating between positive and negative values, the tracking is good, corresponding to a small signal slope. The step size is then reduced.

Mean-Squared Overload Noise Calculation

Consider a sinusoidal input test signal $x(t) = A \sin \theta = A \sin \omega_m t$, where $\theta = \omega_m t$ and $T = 1/f_m$. Let the sampling rate be f_s samples/s and the bandwidth of $x(t)$ be $B = f_m$ Hz.

The maximum slope of $x(t)$ is $2\pi A f_m$ at $t = 0$ and the slope for the delta modulator is $k' f_s$. It can be seen that

$$k' f_s \geq 2\pi A f_m \qquad (17.3)$$

for no overloading. If $k' f_s < 2\pi A f_m$, overload may occur. This is shown in Figure 17.5.

Figure 17.5 Calculation of overload noise with a sinusoidal test signal.

Assume that the step size is small, so that the quantisation noise is negligible. When $\theta > -\theta_1$, overload occurs. At $-\theta_1$, the slope is

$$\frac{dx}{d\theta} = \frac{dx}{dt}\frac{dt}{d\theta} \tag{17.4}$$

where dx/dt is equal to the slope of the delta modulator (gradient of the straight line) and $d\theta/dt = 1/\omega_m = 1/2\pi f_m$. Therefore,

$$\frac{dx}{d\theta} = k' f_s \frac{1}{\omega_m} \tag{17.5}$$

The overload noise N_o is defined as the average of the square of the difference between the signal $x(t)$ and the predictor input

$$N_o = \frac{1}{\pi} \int_{-\theta_1}^{\theta_2} \{A \sin \theta - [\frac{k' f_s}{\omega_m}(\theta + \theta_1) - A \sin \theta_1]\}^2 \, d\theta \tag{17.6}$$

The second term in equation (17.6) corresponds to the straight line in Figure 17.5. Since the overload region occurs twice in each cycle of the sine wave, we need only average over half of the cycle. The overload noise can be found by evaluating equation (17.6) on the assumption of $\theta_1 \leq \pi/4$ and $\theta_2 \simeq 2\theta_1$. It should be noted that equation (17.6) is also valid when the input signal is random in nature.

Mean-Squared Quantisation Noise Calculation

For no overload, the mean-squared variation about the desired level (0 volt) is exactly that calculated for PCM systems. For PCM, the mean-squared quantisation noise is $a^2 / 12$, where a is the spacing between adjacent levels. For delta modulation, the spacing $a = 2k'$ and the mean-squared quantisation noise is

$$N_q = (2k')^2 / 12 = k'^2 / 3 \tag{17.7}$$

For small values of N_o and N_q, we can decouple the noise effects. They are assumed independent of each other and the total mean-squared noise is taken as

$$N = N_o + N_q \tag{17.8}$$

Since the mean-squared output signal power of $x(t)$ is $S_0 = A^2 / 2$, the mean-squared output signal-to-noise ratio is

$$SNR = A^2 / 2N$$

$$= \frac{A^2}{2(N_o + N_q)} \qquad (17.9)$$

References

[1] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw Hill, 1990.

[2] S. Haykin, Communication Systems, 4/e, J. Wiley & Sons, 2001.

[3] L. W. Couch, II, Digital and Analog Communication Systems, 6/e, Prentice Hall, 2001.

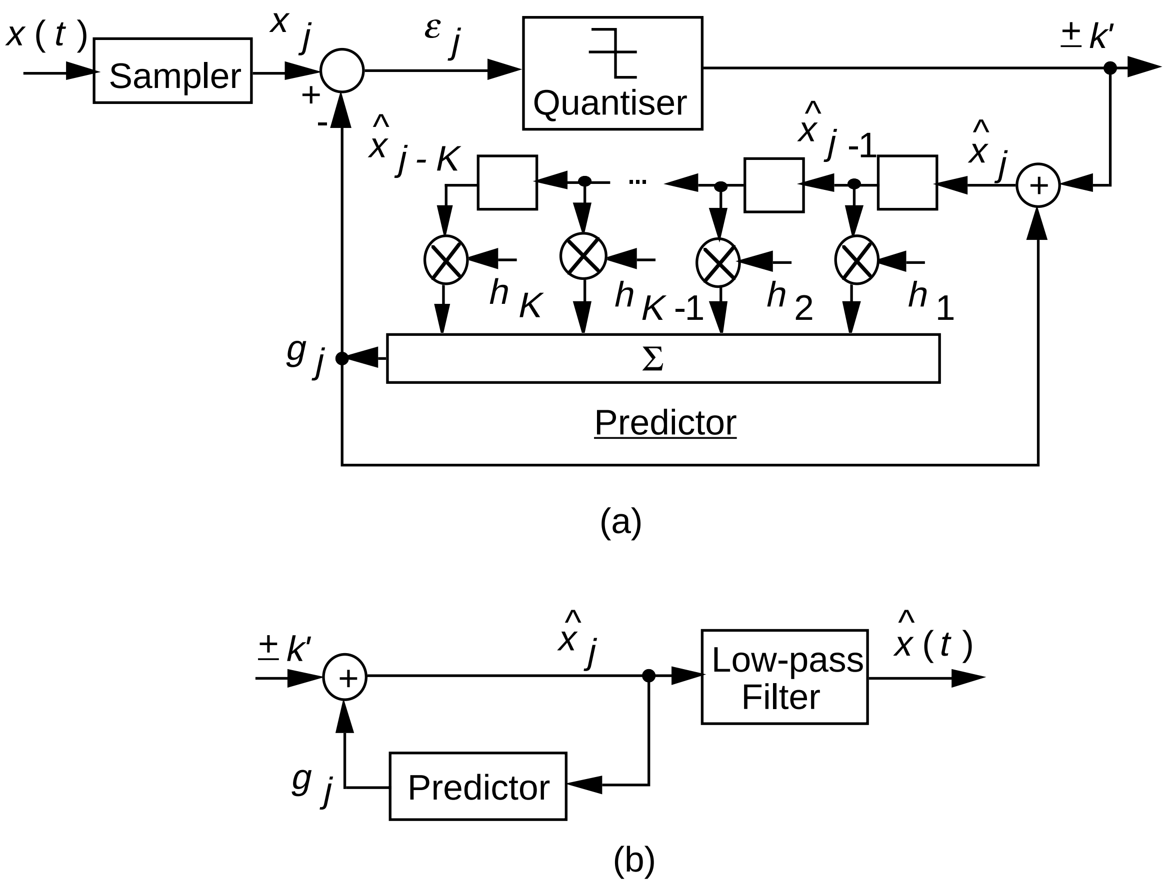

Figure 17.1 (a) Delta modulator. (b) Delta demodulator.

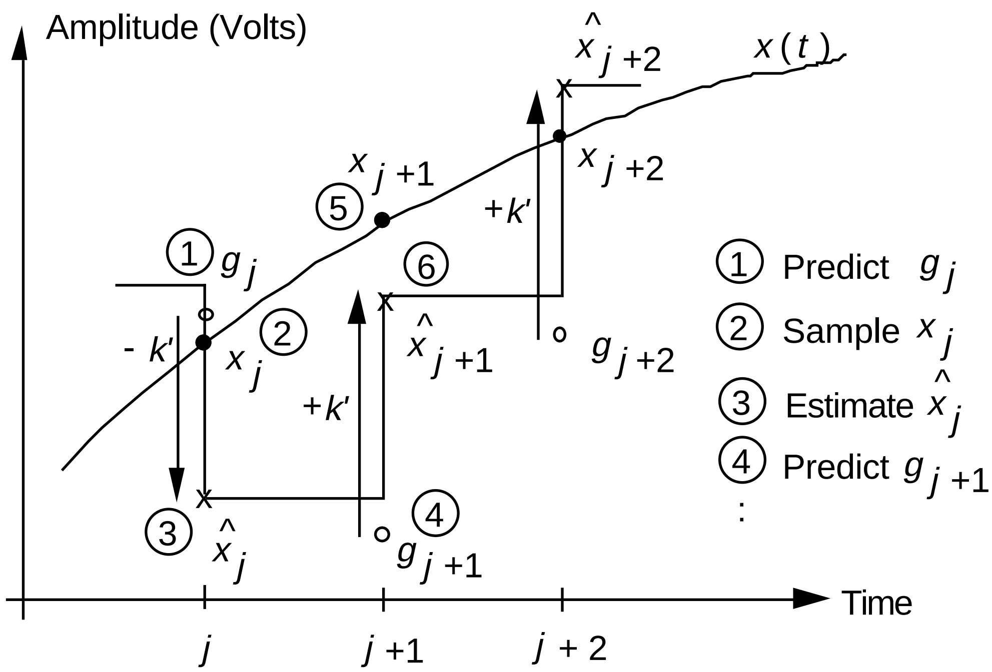

Figure 17.2 Operation of delta modulator with previous-sample prediction.

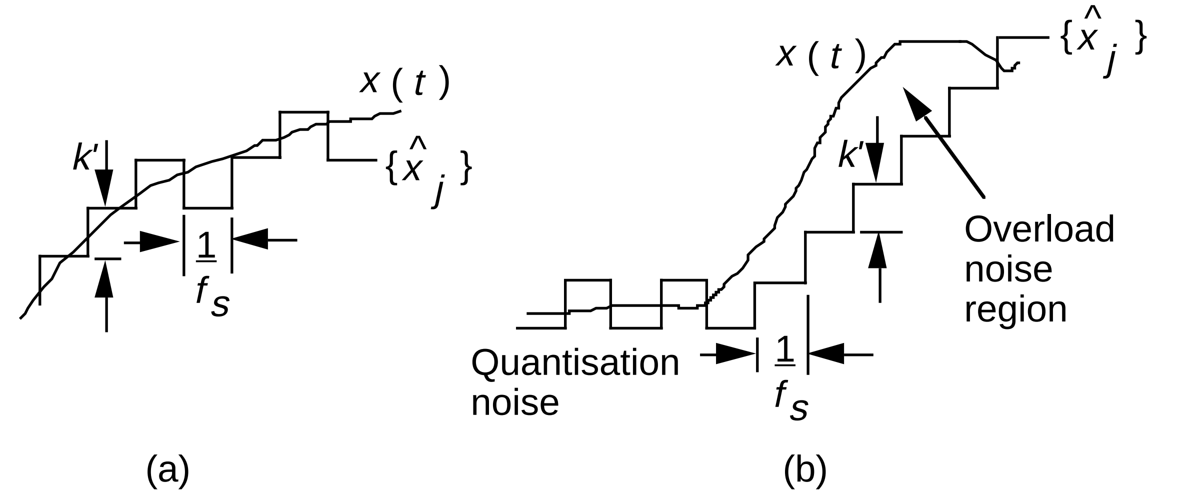

Figure 17.3 (a) Quantisation noise. (b) Overload noise.

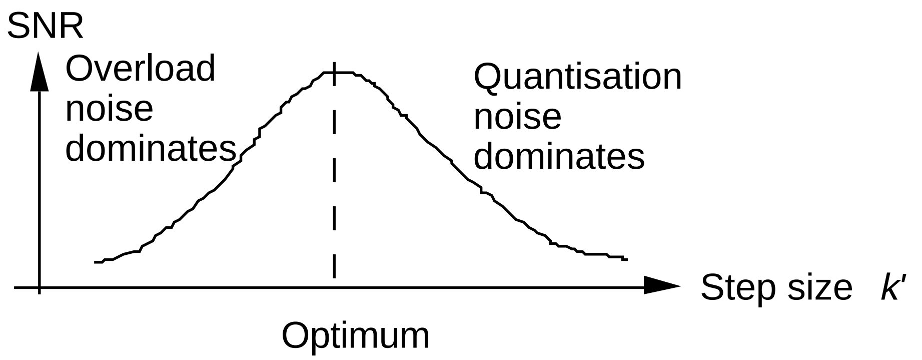

Figure 17.4 Typical performance curve for delta modulation.

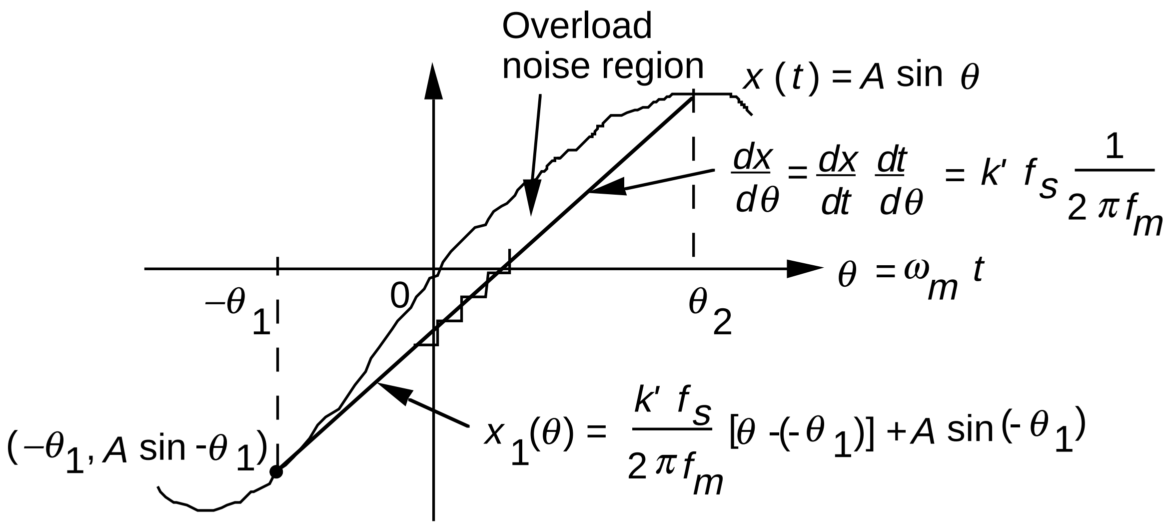

Figure 17.5 Calculation of overload noise with a sinusoidal test signal.