7. Double-Sideband Modulation

Review of Normal Amplitude Modulation

A normal amplitude-modulated signal is given by

$$s_c(t) = [A + m(t)] \cos 2\pi f_c t \quad (7.1)$$

$$s_c(t) = \underbrace{A \cos 2\pi f_c t}_{carrier} + \underbrace{m(t) \cos 2\pi f_c t}_{sidebands} \quad (7.2)$$

where A is a constant, $m(t)$ is the modulating signal, and f_c is the carrier frequency. The *modulation index* m is defined as [1]

$$m = \frac{|\min m(t)|}{A} \quad (7.3)$$

and the efficiency η of a normal AM signal is defined as [2]

$$\eta = \frac{P_s}{P_t} \text{ x } 100\% \quad (7.4)$$

where P_s is the power carried by the sidebands and P_t is the total power of the normal AM signal.

Spectrum of DSB Signals

For double-sideband (DSB) modulation, $A = 0$ and

$$s_c(t) = m(t) \cos 2\pi f_c t \quad (7.5)$$

The Fourier transform of $s_c(t)$ is

$$S_c(f) = \frac{1}{2} [M(f-f_c) + M(f+f_c)] \quad (7.6)$$

Figure 7.1 shows the waveforms and spectra associated with a DSB signal. Clearly, the envelope of the modulated signal does not have the same shape as $m(t)$. As with AM, DSB modulation shifts the spectrum of $m(t)$ to the carrier frequency f_c. The bandwidth of the modulated signal is 2 f_m Hz, where f_m is the bandwidth of the modulating signal $m(t)$.

Figure 7.1 Waveforms and spectra associated with DSB signal.

Generation of DSB Signals

The process of generating a DSB signal is shown in Figure 7.2. DSB modulation can be achieved by using non-linear devices, such as a diode. This is shown in Figure 7.3 [3, 4].

Figure 7.2 Generation of DSB signal.

Figure 7.3 A single-balanced modulator.

Let the input-output characteristic of a diode be approximated by a power series

$$v_o(t) = a\, v_i(t) + b\, v_i^2(t) \qquad (7.7)$$

where a, b are constants. Consider the diode D_1 in the upper portion of the circuit shown in Figure 7.3. The input voltage to the diode D_1 is

$$v_{i,1}(t) = v_i(t) = \cos 2\pi f_c t + m(t) \qquad (7.8)$$

and let the output voltage of the diode be $v_{o,1}(t) = v_o(t)$. Substituting equation (7.8) into (7.7), we have

$$\begin{aligned} v_{o,1}(t) &= a[\cos 2\pi f_c t + m(t)] + b[\cos 2\pi f_c t + m(t)]^2 \\ &= a\, m(t) + b\cos^2 2\pi f_c t + b\, m(t)^2 + \\ &\quad a\cos 2\pi f_c t + 2b\, m(t) \cos 2\pi f_c t \end{aligned} \qquad (7.9)$$

Now, consider the diode D_2 in the lower portion of the circuit shown in Figure 7.3. The input voltage to the diode D_2 is

$$v_{i,2}(t) = v_i(t) = \cos 2\pi f_c t - m(t) \qquad (7.10)$$

and let the output voltage of the diode be $v_{o,2}(t) = v_o(t)$. Substituting equation (7.10) into (7.7), we have

$$\begin{aligned} v_{o,2}(t) &= a[\cos 2\pi f_c t - m(t)] + b[\cos 2\pi f_c t - m(t)]^2 \\ &= -a\, m(t) + b\cos^2 2\pi f_c t + b\, m(t)^2 + \\ &\quad a\cos 2\pi f_c t - 2b\, m(t) \cos 2\pi f_c t \end{aligned} \qquad (7.11)$$

Subtracting equation (7.11) from (7.9), we get

$$v_o(t) = v_{o,1}(t) - v_{o,2}(t) = 2a\ m(t) + 4b\ m(t) \cos 2\pi f_c t \tag{7.12}$$

If we pass this signal through a bandpass filter centred at $\pm f_c$, we obtain

$$v'_o(t) = 4b\ m(t) \cos 2\pi f_c t \tag{7.13}$$

That is, we generate a DSB signal. It can be seen that the circuit is balanced with respect to the carrier signal $s(t)$; however, the modulating signal $m(t)$ still appears at the output of the second transformer T_2. For this reason, the modulator is called a *single-balanced modulator*.

From our earlier study of normal AM, multiplication of a signal by a periodic digital signal involves switching the signal $m(t)$ on and off periodically. This can be accomplished by switching elements controlled by the carrier signal $s(t)$. Figure 7.4 shows a single-balanced shunt-bridge diode modulator, where diodes D_1, D_2 and D_3, D_4 are matched pairs. During the positive half-cycle of $s(t)$, terminal c is positive with respect to d, so all the diodes conduct. Terminals a and b have the same potential and are effectively shorted. $v_o(t)$ is zero. During the negative half-cycle of $s(t)$, terminal c is negative with respect to d, all the diodes are open. $v_o(t) = v_i(t)$. $m(t)$ is effectively multiplied by a non-negative periodic signal. Therefore, the component $m(t)$ appears at the input to the bandpass filter. If we pass the signal $v_o(t)$ through a bandpass filter centred at $\pm f_c$, we generate a DSB signal.

Figure 7.4 A single-balanced shunt-bridge diode modulator.

A DSB signal can also be obtained by multiplying $m(t)$ by any periodic digital signal $s(t)$. The modulator is called a *switching modulator* [2]. If we take a periodic square waveform $s(t)$ of period $T_c = 1/f_c$, amplitude $\pm A_c/2$, and pulse width of T_c, the trigonometric Fourier series of $s(t)$ is

$$s(t) = \frac{2}{T_c} \sum_{n=1}^{\infty} \left(0.5 A_c T_c \frac{\sin n\pi/2}{n\pi/2}\right) \cos 2\pi n f_c t \tag{7.14}$$

$$s(t) = \frac{2}{T_c} \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t \tag{7.15}$$

where $c_n = 0.5 A_c T_c \frac{\sin n\pi/2}{n\pi/2}$. The corresponding complex Fourier series is

$$s(t) = \frac{1}{T_c} \sum_{n=-\infty}^{\infty} \left(0.5 A_c T_c \frac{\sin n\pi/2}{n\pi/2}\right) e^{j2\pi n f_c t} \tag{7.16}$$

Figure 7.5 shows the periodic square waveform and its line spectrum.

Figure 7.5 (a) A periodic square waveform, and (b) its line spectrum.

If the input signal is $v_i(t) = m(t)$, the product of $v_i(t)$ and $s(t)$ is

$$\begin{aligned} v_o(t) &= v_i(t)\, s(t) \\ &= m(t)\, s(t) \\ &= m(t) \frac{2}{T_c} \sum_{n=1}^{\infty} c_n \cos 2\pi n f_c t \end{aligned} \qquad (7.17)$$

$v_o(t)$ consists of an infinite number of DSB signals at carrier frequencies f_c, $2f_c$, $3f_c$, ... If we pass the signal $v_o(t)$ through a bandpass filter centred at $\pm f_c$, the filtered signal is

$$v'_o(t) = \frac{2}{T_c} c_1\, m(t) \cos 2\pi f_c t \qquad (7.18)$$

That is, we obtain a DSB signal. Figure 7.6 shows the waveforms and spectra associated with a switching modulator.

Figure 7.6 Waveforms and spectra associated with a switching modulator.

It is possible to design a balanced modulator such that the input to the bandpass filter does not contain the message signal $m(t)$ or the carrier signal $s(t)$. A circuit balanced with respect to both input signals is called a *double-balanced modulator*. Figure 7.7 shows a double-balanced modulator, known as a *ring modulator* [4, 5].

Figure 7.7 A ring modulator.

During the positive half-cycle of $s(t)$, diodes D_1 and D_3 conduct, and D_2 and D_4 are open. Terminal a is connected to c, terminal b is connected to d, and $v_o(t)$ is proportional to $m(t)$. During the negative half-cycle of $s(t)$, diodes D_1 and D_3 are open, and diodes D_2 and D_4 are conducting. Terminal a is connected to d, terminal b is connected to c, and $v_o(t)$ is proportional to $-m(t)$. $m(t)$ is effectively multiplied by a sinusoidal carrier waveform. Therefore, the components $m(t)$ and $s(t)$ do not appear at the input to the bandpass filter. If we pass the signal $v_o(t)$ through a bandpass filter centred at $\pm f_c$, we generate a DSB signal.

Demodulation of DSB Signals [3]

Since the envelope of the modulated signal does not have the same shape as $m(t)$, an envelope detector cannot be used to recover the message signal. Demodulation of DSB signals can be accomplished by using a synchronous detector. This is shown in Figure 7.8.

Figure 7.8 Synchronous detector.

Let $s_c(t)$ be the input signal to the synchronous detector. At the receiving end, the bandpass signal is multiplied by a locally generated carrier signal cos $2\pi f_c t$, which is in synchronism with the transmitted carrier signal. The output of the multiplier is

$$\begin{aligned} x(t) &= m(t) \cos 2\pi f_c t \cos 2\pi f_c t \\ &= 0.5\, m(t) + 0.5 m(t) \cos 4\pi f_c t \end{aligned} \quad (7.19)$$

If we suppress the last term by a low-pass filter, we get

$$y(t) = 0.5m(t) \quad (7.20)$$

That is, we can recover the component $m(t)$. If the carrier signal shifts in frequency or phase, the resultant signal is distorted or attenuated.

Carrier Recovery

For the demodulation of a DSB signal, we can use a squaring loop to generate a local carrier signal. Figure 7.9 shows a carrier-recovery squaring loop for a DSB signal [2].

Figure 7.9 Carrier-recovery squaring loop for DSB signal.

However, the squaring loop has a disadvantage: there is a 180^o phase ambiguity. If the transmitted carrier signal is shifted by 180^o, the demodulated signal is $-m(t)$. The demodulated signal is inverted. To remove the phase ambiguity, we can send a known test signal or use *differential coding* and *decoding*.

Quadrature Amplitude Modulation (QAM) [5]

We have seen that DSB signals require a transmission bandwidth equal to twice the bandwidth of the message signal $m(t)$. To increase the transmission bandwidth efficiency, it is possible to send two DSB signals using carriers of the same frequency but in phase quadrature. Both modulated signals occupy the same frequency band. Yet they can be separated at the receiver by synchronous detection using two local carriers in phase quadrature. The technique is known as *Quadrature Amplitude Modulation (QAM)* or *quadrature multiplexing* and the arrangement is shown in Figure 7.10.

Figure 7.10 A QAM transmitter and receiver.

A QAM signal is given by

$$s_c(t) = m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t \tag{7.21}$$

At the receiving end, the modulated signal is multiplied by two carriers in phase quadrature. The signals at the outputs of the multipliers are

$$\begin{aligned} x_1(t) &= 2\, s_c(t) \cos 2\pi f_c t \\ &= m_1(t) + m_1(t) \cos 4\pi f_c t + m_2(t) \sin 4\pi f_c t \end{aligned} \tag{7.22}$$

and

$$\begin{aligned} x_2(t) &= 2\, s_c(t) \sin 2\pi f_c t \\ &= m_2(t) - m_2(t) \sin 4\pi f_c t + m_1(t) \sin 4\pi f_c t \end{aligned} \tag{7.23}$$

If we suppress the high-frequency components by low-pass filters, we get

$$y_1(t) = m_1(t) \tag{7.24}$$

and

$$y_2(t) = m_2(t) \tag{7.25}$$

That is, the desired outputs are obtained. Supposing that the local carrier signal is $\cos (2\pi f_c t + \phi_0)$, then the multiplier output in the upper portion of the circuit becomes

$$\begin{aligned} x_1(t) &= 2\, s_c(t) \cos (2\pi f_c t + \phi_0) \\ &= m_1(t)\cos \phi_0 + m_1(t)\cos (4\pi f_c t + \phi_0) - \\ &\quad m_2(t)\sin \phi_0 + m_2(t) \sin (4\pi f_c t + \phi_0) \end{aligned} \tag{7.26}$$

If we suppress the second and the last terms by a low-pass filter, we get

$$y_1(t) = m_1(t)\cos \phi_0 + m_2(t)\sin \phi_0 \tag{7.27}$$

The desired signal $m_1(t)$ and the unwanted signal $m_2(t)$ appear in the upper portion of the circuit. Also, it can be shown that $y_2(t)$ contains the desired signal $m_2(t)$ and the unwanted signal $m_1(t)$. Modulated signals having the same carrier frequency now interfere with each other. This is called *cochannel interference* and must be avoided. Worse problems arise when the local carrier frequency is in error. Therefore, the local carrier must not only be of the same frequency but must be synchronised in phase with the carrier signal. A slight error in the frequency or the phase of the local carrier signal will not only result in loss and distortion of signals, but will also lead to interference.

Quadrature multiplexing is used in colour television to multiplex the signals which carry the information about colours.

Frequency Division Multiplexing (FDM)

One of the basic problems in communication engineering is the design of a system which allows many individual signals from users to be transmitted simultaneously over a single communication channel. The most common method is to translate individual signals from one frequency region to another frequency region. Suppose that we have several different signals of the same bandwidth. If we translate each one of the signals to a different frequency region such that the translated signal spectra do not overlap each other, then all these signals can now be transmitted along a single communication channel. At the receiving end, the signals can be separated and recovered. We now have a frequency-multiplexed system. Such a multiplexing technique is called *frequency division multiplexing (FDM)*. Frequency translation can be accomplished by multiplying a low-frequency modulating signal with a high-frequency sinusoidal carrier signal. Figure 7.11 shows the transmitter, the receiver, and the spectrum of a 5-user FDM system with carrier frequencies $f_{c1} < f_{c2} < ... < f_{c5}$.

Figure 7.11 FDM system.

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[2] L. W. Couch II, Digital and Analog Communication Systems, 5/e, Prentice Hall, 1997.

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[5] B. P. Lathi, Modern Digital and Analog Communication Systems, 3/e, Oxford University Press, 1998.

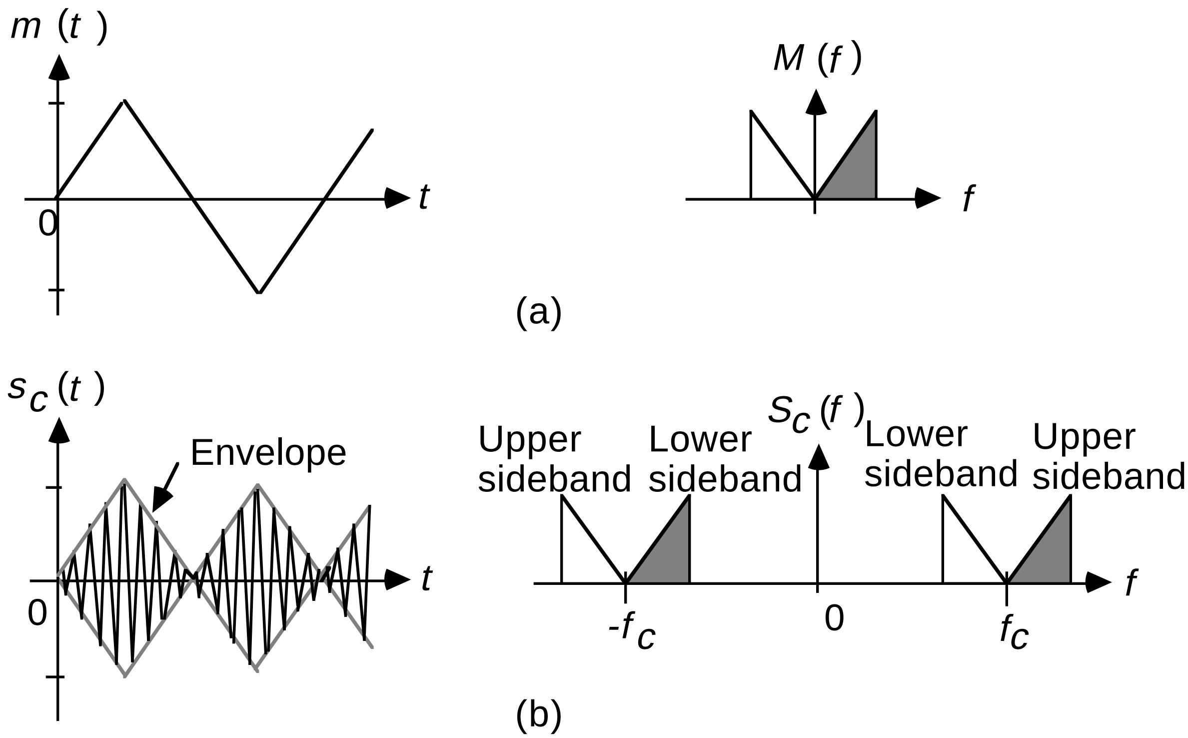

Figure 7.1 Waveforms and spectra associated with DSB signal.

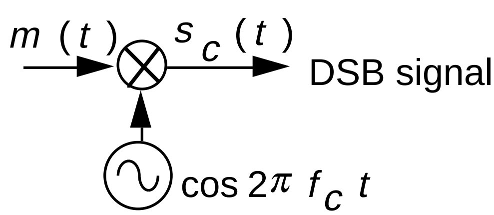

Figure 7.2 Generation of DSB signal.

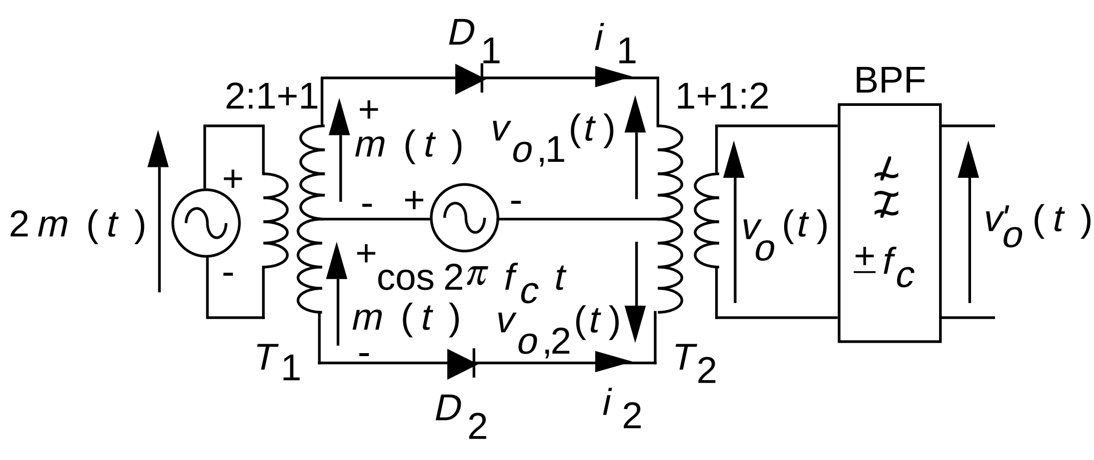

Figure 7.3 A single-balanced modulator.

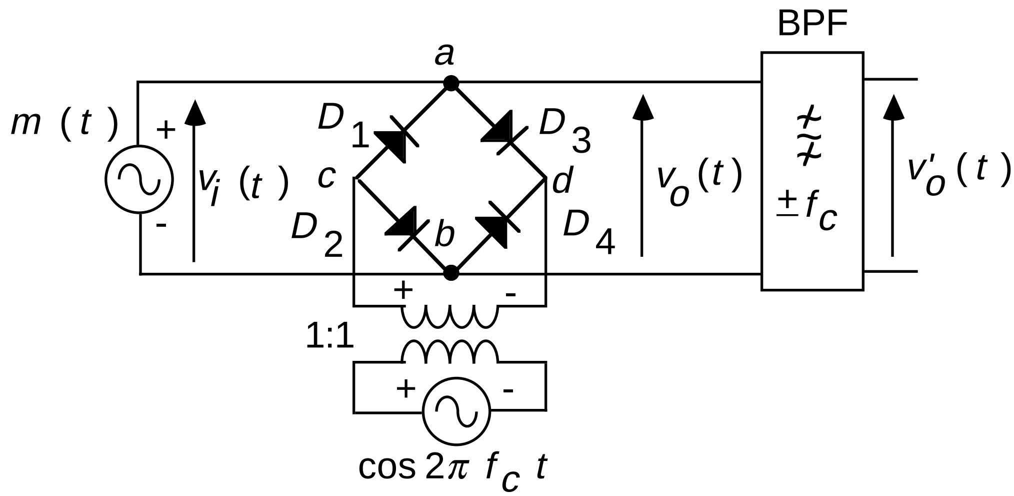

Figure 7.4 A single-balanced shunt-bridge diode modulator.

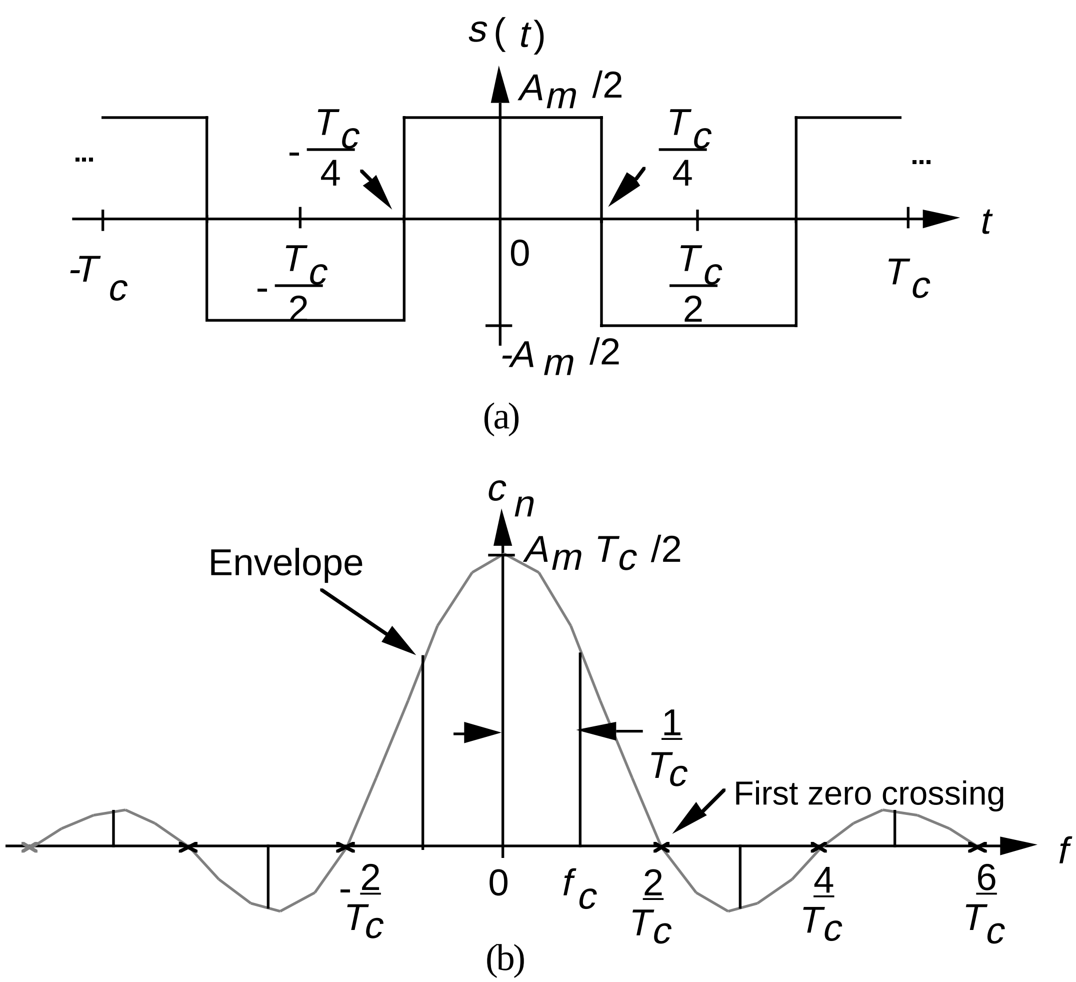

Figure 7.5 (a) A periodic square waveform, and (b) its line spectrum.

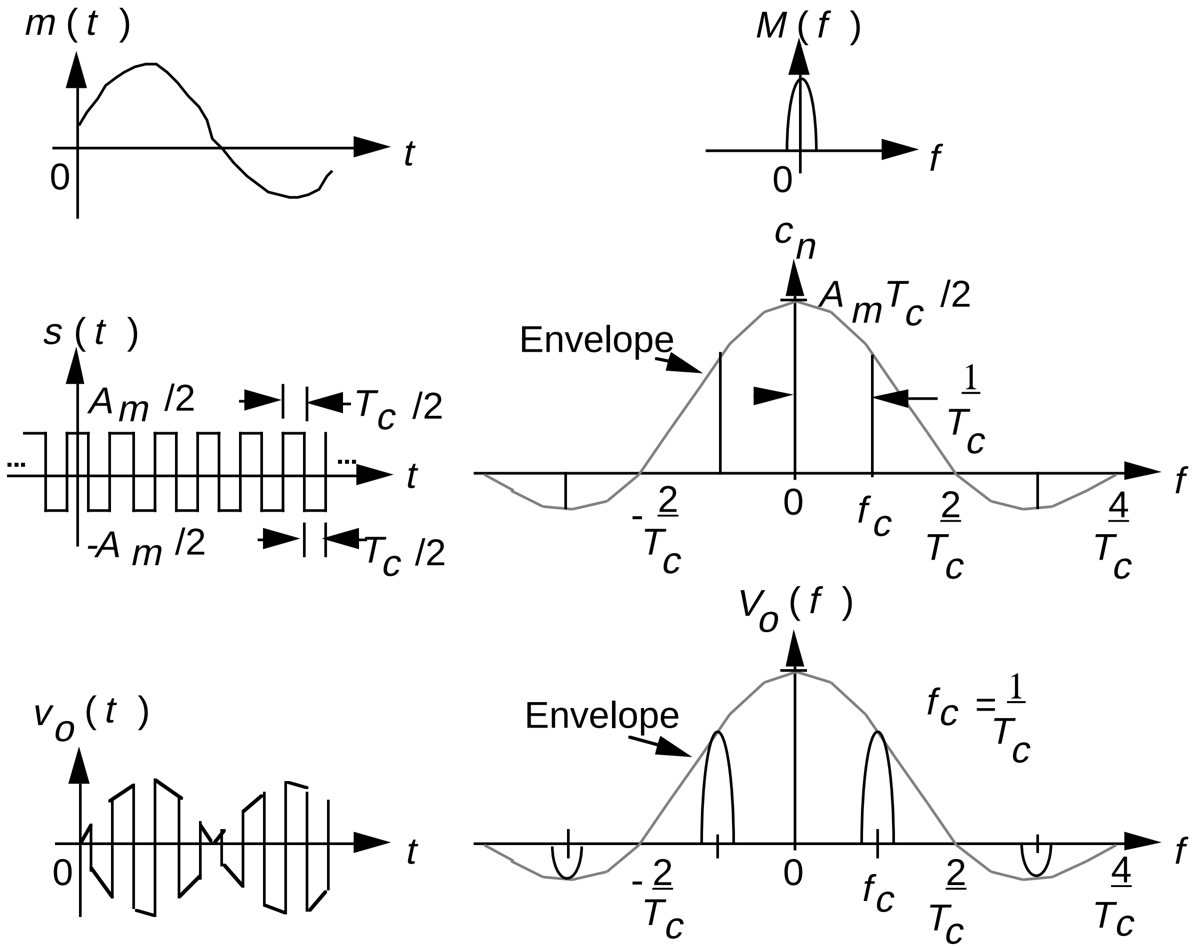

Figure 7.6 Waveforms and spectra associated with a switching modulator.

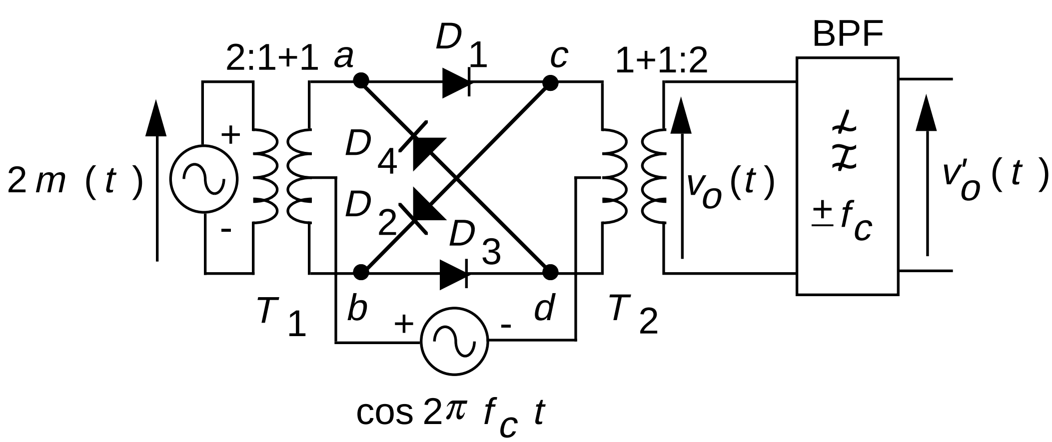

Figure 7.7 A double-balanced ring modulator.

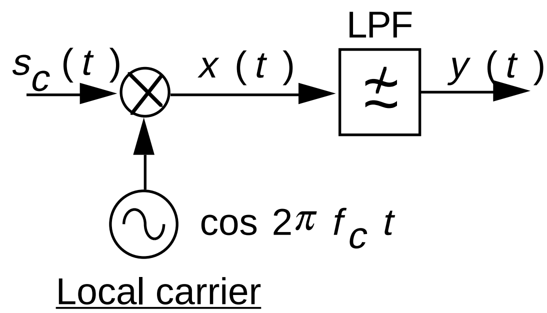

Figure 7.8 Synchronous detector.

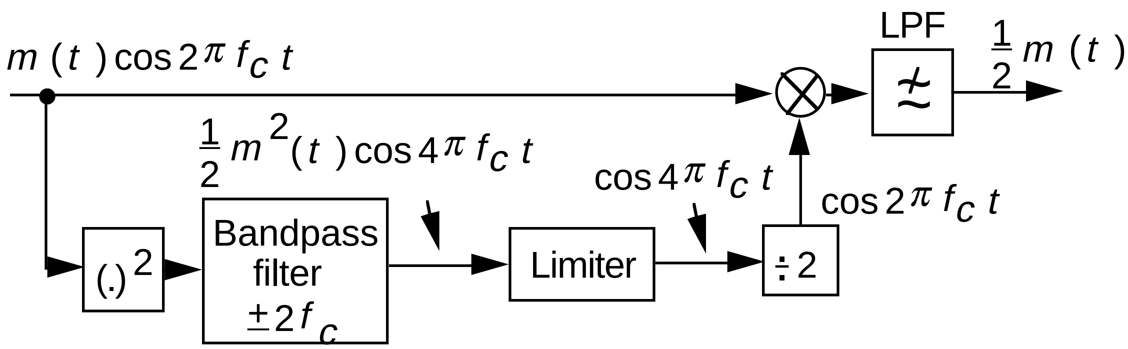

Figure 7.9 Carrier-recovery squaring loop for DSB signal.

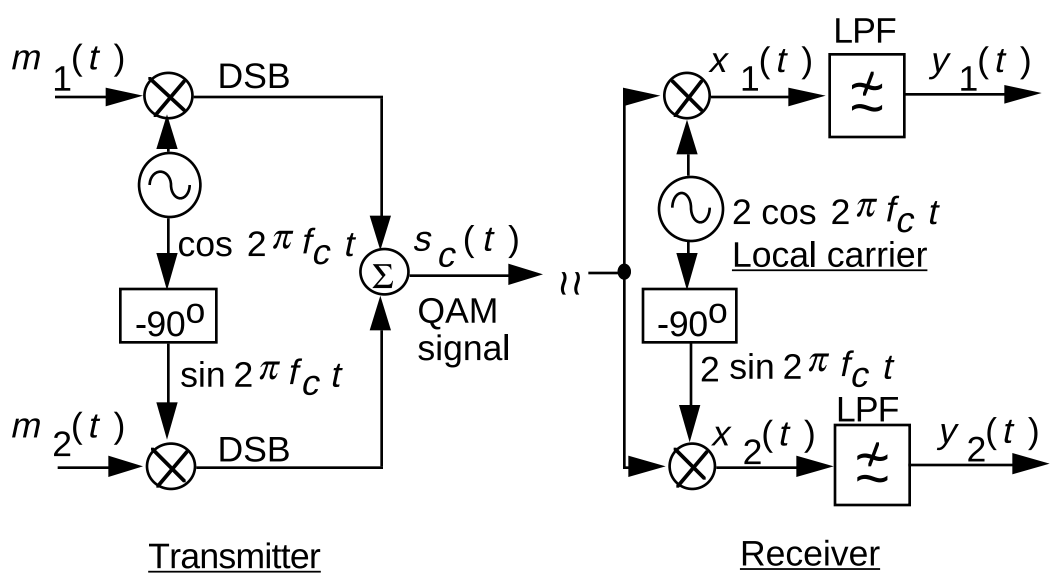

Figure 7.10 A QAM transmitter and receiver.

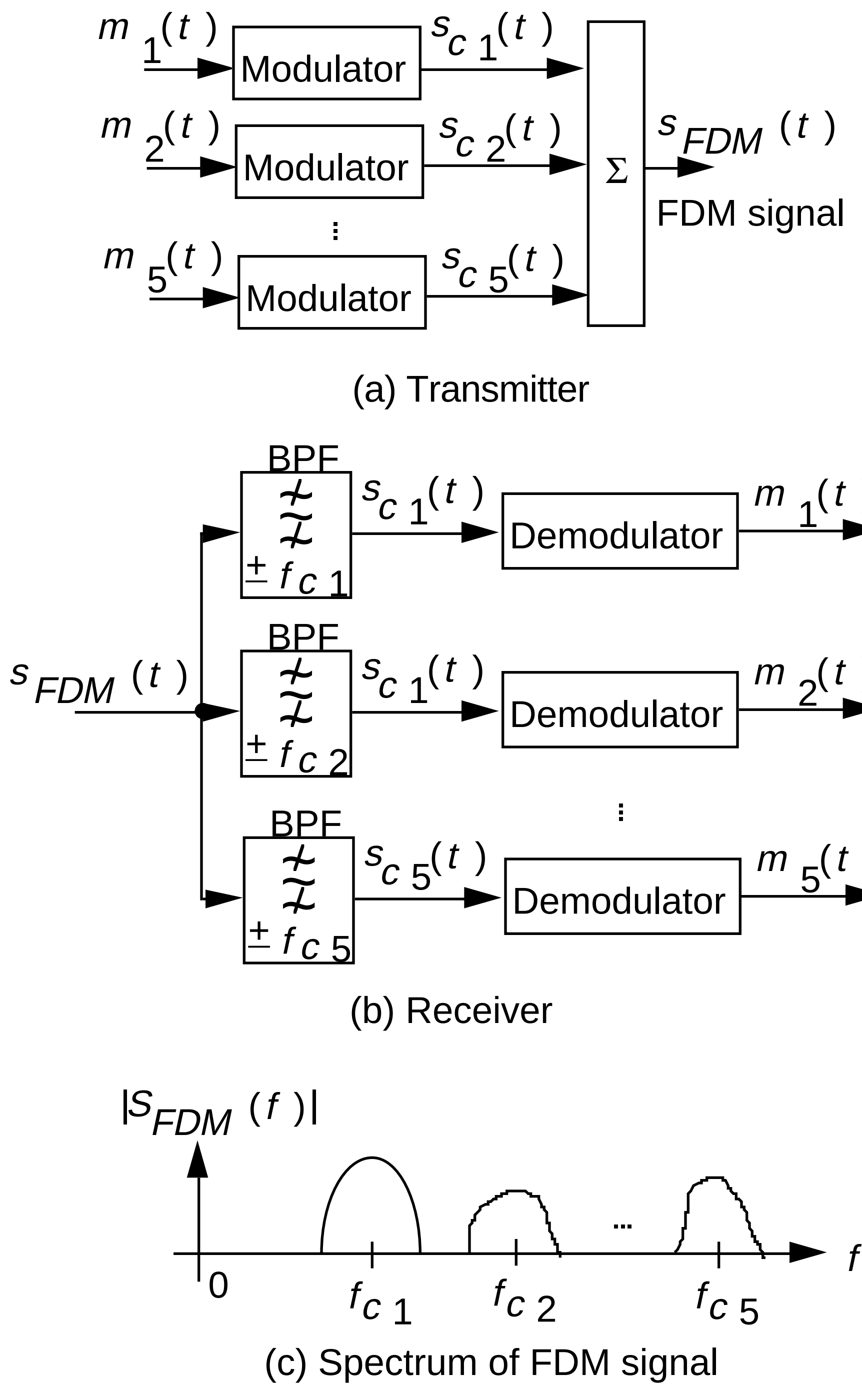

Figure 7.11 FDM system.