

9. Angle Modulation

Angle modulation encompasses *phase modulation (PM)* and *frequency modulation (FM)*. The phase angle of a sinusoidal carrier signal is varied according to the modulating signal. In angle modulation, the spectral components of the modulated signal are not related in a simple fashion to the spectrum of the modulating signal. Superposition does not apply and the bandwidth of the modulated signal is usually much greater than the modulating signal bandwidth.

Definitions

A bandpass signal is represented by

$$s_c(t) = A(t) \cos \theta(t) \quad (9.1)$$

where $A(t)$ is the envelope and $\theta(t) = \omega_c t + \phi(t) = 2\pi f_c t + \phi(t)$. For angle modulation, we can write

$$s_c(t) = A \cos [2\pi f_c t + \phi(t)] \quad (9.2)$$

where A is a constant and $\phi(t)$ is a function of the modulating signal. $\phi(t)$ is called the *instantaneous phase deviation* of $s_c(t)$.

The *instantaneous angular frequency* of $s_c(t)$ is defined as [1]

$$\omega_i(t) = \frac{d\theta(t)}{dt} \quad (9.3)$$

In terms of frequency, the *instantaneous frequency* of $s_c(t)$ is [2]

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \quad (9.4)$$

$$= f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt} \quad (9.5)$$

$\frac{1}{2\pi} \frac{d\phi(t)}{dt}$ is known as the *instantaneous frequency deviation*. The *peak (maximum) frequency deviation* is [1]

$$\Delta f = \max \left| \frac{1}{2\pi} \frac{d\phi(t)}{dt} \right| = \max |f_i(t) - f_c| \quad (9.6)$$

Phase Modulation

For PM, the instantaneous phase deviation is proportional to the modulating signal $m_p(t)$:

$$\phi(t) = k_p m_p(t) \quad (9.7)$$

where k_p is a constant. Thus, a phase-modulated signal is represented by

$$s_c(t) = A \cos [2\pi f_c t + k_p m_p(t)] \quad (9.8)$$

Substituting equation (9.7) into (9.5), the *instantaneous frequency* of $s_c(t)$ can be written as

$$f_i(t) = f_c + \frac{1}{2\pi} k_p \frac{dm_p(t)}{dt} \quad (9.9)$$

The *peak (maximum) phase deviation* is [2]

$$\Delta\phi = \max |\phi(t)| \quad (9.10)$$

$$= k_p \max |m_p(t)| \quad (9.11)$$

The *phase modulation index* is given by [2]

$$\beta_p = \Delta\phi \quad (9.12)$$

Frequency Modulation

For FM, the instantaneous frequency deviation is proportional to the modulating signal $m_f(t)$:

$$\frac{d\phi(t)}{dt} = k_f m_f(t) \quad (9.13)$$

where k_f is a constant, and

$$\phi(t) = k_f \int_{-\infty}^t m_f(\tau) d\tau + \phi(-\infty) \quad (9.14)$$

$\phi(-\infty)$ is usually set to 0. Thus, a frequency-modulated signal is represented by

$$s_c(t) = A \cos [2\pi f_c t + k_f \int_{-\infty}^t m_f(\tau) d\tau] \quad (9.15)$$

Substituting equation (9.13) into (9.5), the *instantaneous frequency* of $s_c(t)$ can be written as

$$f_i(t) = f_c + \frac{1}{2\pi} k_f m_f(t) \quad (9.16)$$

Figure 9.1 shows the modulating signal $m_f(t)$, the instantaneous frequency $f_i(t)$, and the FM signal when a sawtooth signal is used as a modulating signal.

Figure 9.1 Frequency modulation: (a) Modulating signal, (b) instantaneous frequency, and (c) FM signal.

The *frequency deviation* from the carrier frequency is [2]

$$f_d(t) = f_i(t) - f_c = \frac{1}{2\pi} \frac{d\phi(t)}{dt} = \frac{1}{2\pi} k_f m_f(t) \quad (9.17)$$

and the *peak frequency deviation* is [2]

$$\Delta f = \max \left| \frac{1}{2\pi} \frac{d\phi(t)}{dt} \right| \quad (9.18)$$

$$= \frac{1}{2\pi} k_f \max |m_f(t)| \quad (9.19)$$

Generation of Angle-Modulated Signal [2]

It can be seen from equations (9.7) and (9.14) that PM and FM differ only by a possible integration or differentiation of the modulating signal. From equations (9.7) and (9.14), we obtain

$$m_f(t) = \frac{k_p}{k_f} \frac{dm_p(t)}{dt} \quad (9.20)$$

and

$$m_p(t) = \frac{k_f}{k_p} \int_{-\infty}^t m_f(\tau) d\tau \quad (9.21)$$

If we differentiate the modulating signal $m_p(t)$ and frequency-modulate using the differentiated signal, we get a PM signal. On the other hand, if we integrate the modulating

signal $m_f(t)$ and phase-modulate using the integrated signal, we get a FM signal. Therefore, we can generate a PM signal using a frequency modulator or we can generate a FM signal using a PM modulator. This is shown in Figure 9.2.

Figure 9.2 Generation of (a) PM using a frequency modulator, and (b) FM using a phase modulator.

Spectrum of an Angle-Modulated Signal [1]

For angle modulation,

$$s_c(t) = A \cos [2\pi f_c t + \phi(t)] \quad (9.22)$$

and we can write

$$s_c(t) = \text{Re} \{Ae^{j[2\pi f_c t + \phi(t)]}\} = \text{Re} \{Ae^{j2\pi f_c t} e^{j\phi(t)}\} \quad (9.23)$$

Expanding $e^{j\phi(t)}$ in a power series yields

$$s_c(t) = \text{Re} \{Ae^{j2\pi f_c t} [1 + j\phi(t) - \frac{\phi^2(t)}{2!} - \dots + j^n \frac{\phi^n(t)}{n!} + \dots]\} \quad (9.24)$$

$$= A[\cos 2\pi f_c t - \phi(t)\sin 2\pi f_c t - \frac{\phi^2(t)}{2!} \cos 2\pi f_c t + \frac{\phi^3(t)}{3!} \sin 2\pi f_c t + \dots] \quad (9.25)$$

It can be seen that the spectrum of an angle-modulated signal consists of an unmodulated carrier plus spectra of $\phi(t)$, $\phi^2(t)$, ..., and is not related to the spectrum of the modulating signal in a simple fashion.

Narrowband Angle Modulation [1]

If $\max |\phi(t)| \ll 1$, we can neglect all higher-power terms of $\phi(t)$ in equation (9.25) and we have a narrowband angle-modulated signal

$$s_c(t) \approx A[\cos 2\pi f_c t - \phi(t)\sin 2\pi f_c t] \quad (9.26)$$

For PM,

$$s_c(t) \approx A[\cos 2\pi f_c t - k_p m_p(t)\sin 2\pi f_c t] \quad (9.27)$$

For FM,

$$s_c(t) \approx A \left\{ \cos 2\pi f_c t - \left[k_f \int_{-\infty}^t m_f(\tau) d\tau \right] \sin 2\pi f_c t \right\} \quad (9.28)$$

Narrowband Frequency Modulation [3]

Because of the difficulty of analysing general angle-modulated signals, we shall only consider a sinusoidal modulating signal. Let the modulating signal of a narrowband FM signal be

$$m_f(t) = a_m \cos 2\pi f_m t \quad (9.29)$$

Substituting (9.29) into (9.14), we have

$$\phi(t) = \frac{k_f a_m}{2\pi f_m} \sin 2\pi f_m t \quad (9.30)$$

$$= \beta_f \sin 2\pi f_m t \quad (9.31)$$

where $\beta_f = k_f a_m / (2\pi f_m)$. β_f is called the *frequency modulation index* and β_f is only defined for a sinusoidal modulating signal. Differentiating (9.31) and substituting $\frac{d\phi(t)}{dt}$ into (9.18), we have

$$\beta_f = \frac{\Delta f}{B} \quad (9.32)$$

where $B = f_m$ is the bandwidth of the modulating signal. Substituting equation (9.31) into (9.2), we have

$$s_c(t) = A \cos (2\pi f_c t + \beta_f \sin 2\pi f_m t) \quad (9.33)$$

$$= A [\cos 2\pi f_c t \cos (\beta_f \sin 2\pi f_m t) - \sin 2\pi f_c t \sin (\beta_f \sin 2\pi f_m t)] \quad (9.34)$$

For $\beta_f \ll \pi/2$, $\cos (\beta_f \sin 2\pi f_m t) \approx 1$, $\sin (\beta_f \sin 2\pi f_m t) \approx \beta_f \sin 2\pi f_m t$, and

$$s_c(t) \approx A [\cos 2\pi f_c t - \beta_f \sin 2\pi f_c t \sin 2\pi f_m t] \quad (9.35)$$

$$\approx A \cos 2\pi f_c t - \frac{\beta_f A}{2} [\cos 2\pi (f_c - f_m)t - \cos 2\pi (f_c + f_m)t] \quad (9.36)$$

$$\approx \text{Re} \left[e^{j2\pi f_c t} \left(A - \frac{\beta_f A}{2} e^{-j2\pi f_m t} + \frac{\beta_f A}{2} e^{j2\pi f_m t} \right) \right] \quad (9.37)$$

Equation (9.36) contains the carrier term plus two sideband terms. The **bandwidth** of the narrowband FM signal is $2f_m$ Hz.

In the AM case with sinusoidal modulating signal $m(t) = a_m \cos 2\pi f_m t$,

$$s_c(t) = [A + a_m \cos 2\pi f_m t] \cos 2\pi f_c t \quad (9.38)$$

$$s_c(t) = A \cos 2\pi f_c t + \frac{a_m}{2} [\cos 2\pi(f_c - f_m)t + \cos 2\pi(f_c + f_m)t] \quad (9.39)$$

$$s_c(t) = \text{Re}[e^{j2\pi f_c t} (A + \frac{mA}{2} e^{-j2\pi f_m t} + \frac{mA}{2} e^{j2\pi f_m t})] \quad (9.40)$$

where the modulation index $m = \frac{a_m}{A}$. Figure 9.3 shows the vector representation of a narrowband FM signal and an AM signal.

Figure 9.3 Vector representation of (a) narrowband FM, and (b) AM.

It can be seen that the resultant of the two sideband vectors in the FM case is always in phase quadrature with the unmodulated carrier, whereas the resultant of the two sideband vectors in the AM case is always in phase with the unmodulated carrier. The distinction and similarity between narrowband FM (or phase modulation) leads us to a commonly used method of generating narrowband angle-modulated signals.

Generation of Narrowband PM and Narrowband FM

The generation of narrowband PM and narrowband FM signals is easily accomplished in view of equations (9.27) and (9.28). This is shown in Figure 9.4.

Figure 9.4 Generation of (a) narrowband PM, and (b) narrowband FM.

References

- [1] H. P. Hsu, Analog and Digital Communications, McGraw-Hill, 1993.
- [2] L. W. Couch II, Digital and Analog Communication Systems, 5/e, Prentice Hall, 1997.
- [3] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw-Hill, 1990.

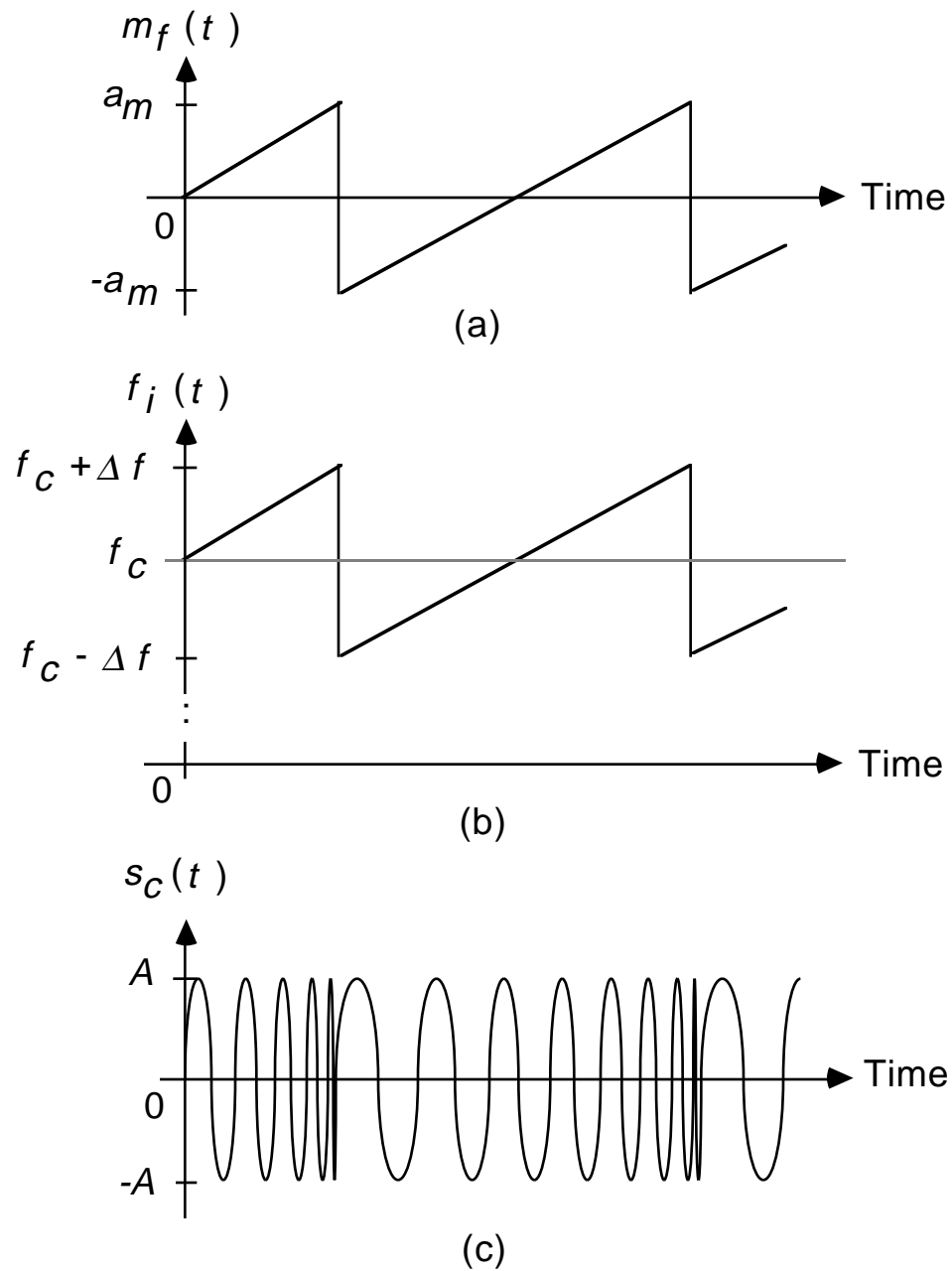


Figure 9.1 Frequency modulation: (a) Modulating signal, (b) instantaneous frequency, and (c) FM signal.

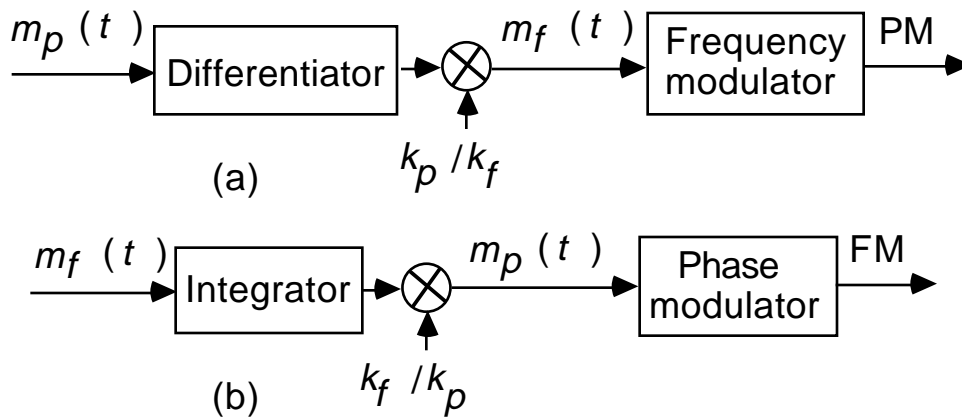


Figure 9.2 Generation of (a) PM using a frequency modulator, and (b) FM using a phase modulator.

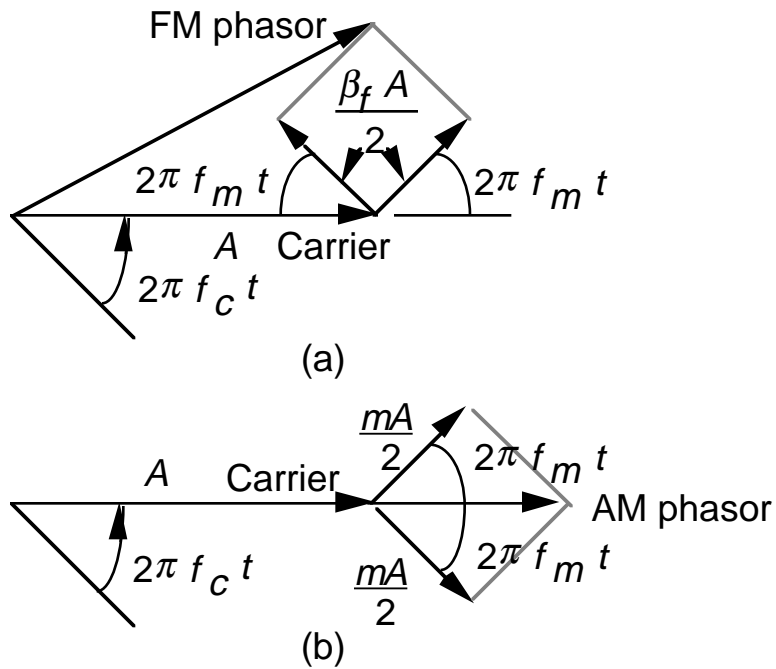


Figure 9.3 Vector representation of (a) narrowband FM, and (b) AM.

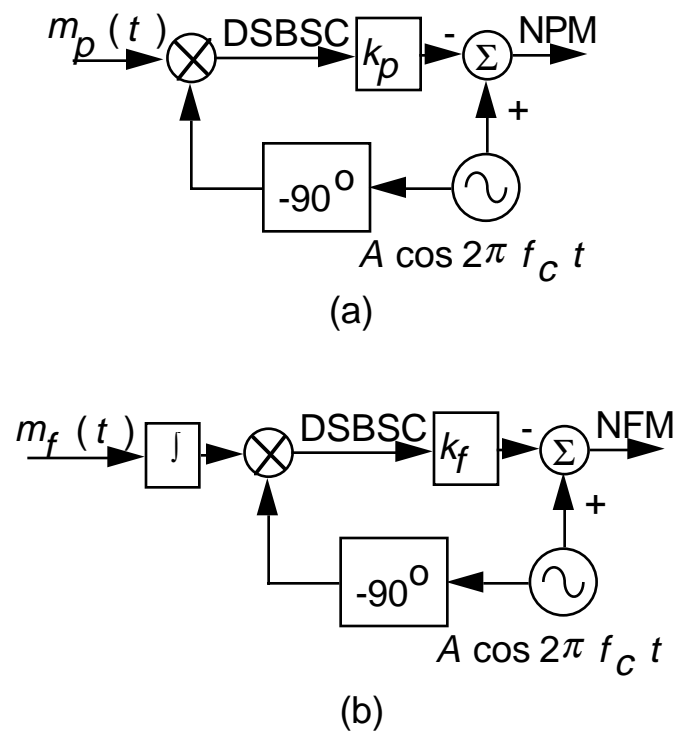


Figure 9.4 Generation of (a) narrowband PM, and (b) narrowband FM.