11. Angle Demodulation

Review of Angle Modulation

Angle modulation encompasses phase modulation (PM) and frequency modulation (FM). We have seen that an angle-modulated signal can be represented by [1]

$$s_c(t) = A\cos\theta(t) \qquad (11.1)$$

$$s_c(t) = A\cos[2\pi f_c t + \phi(t)] \qquad (11.2)$$

where A is a constant, $\theta(t) = 2\pi f_c t + \phi(t)$, and f_c is the carrier frequency. $\phi(t)$ is a function of the modulating signal $m(t)$ and is given by

$$\phi(t) = \begin{cases} k_p m(t) & \text{for } PM \\ k_f \int_{-\infty}^{t} m(\lambda)d\lambda & \text{for } FM \end{cases} \qquad (11.3)$$

and

$$\frac{d\phi(t)}{dt} = \begin{cases} k_p \frac{dm(t)}{dt} & \text{for } PM \\ k_f m(t) & \text{for } FM \end{cases} \qquad (11.4)$$

The instantaneous angular frequency of $s_c(t)$ is defined as [1]

$$\omega_i(t) = \frac{d\theta(t)}{dt} \qquad (11.5)$$

In terms of frequency, the instantaneous frequency of $s_c(t)$ is [2]

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \qquad (11.6)$$

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt} \qquad (11.7)$$

$$f_i(t) = \begin{cases} f_c + \frac{1}{2\pi}k_p \frac{dm(t)}{dt} & \text{for } PM \\ f_c + \frac{1}{2\pi}k_f m(t) & \text{for } FM \end{cases} \qquad (11.8)$$

$\frac{1}{2\pi}\frac{d\phi(t)}{dt}$ is known as the instantaneous frequency deviation.

Angle Demodulation

Demodulation of an angle-modulated signal requires a circuit that produces an output proportional to the instantaneous frequency $f_i(t)$ or the instantaneous frequency deviation $\frac{1}{2\pi}\frac{d\phi(t)}{dt}$ of the input signal to the demodulator.

Frequency Discrimination.

Frequency discrimination is a frequency-to-amplitude conversion process. Consider an angle-modulated signal

$$s_c(t) = A \cos \theta(t) \tag{11.9}$$

where

$$\theta(t) = 2\pi f_c t + \phi(t) \tag{11.10}$$

$$\phi(t) = \begin{cases} k_p m(t) & \text{for } PM \\ k_f \int_{-\infty}^{t} m(\lambda)\,d\lambda & \text{for } FM \end{cases} \tag{11.11}$$

and

$$\frac{d\phi(t)}{dt} = \begin{cases} k_p \frac{dm(t)}{dt} & \text{for } PM \\ k_f m(t) & \text{for } FM \end{cases} \tag{11.12}$$

If we differentiate equation (11.9), we get

$$\begin{aligned} s'_c(t) &= -A \sin \theta(t) \frac{d\theta(t)}{dt} \\ &= -A \sin \theta(t) \left[2\pi f_c + \frac{d\phi(t)}{dt}\right] \\ &= -\underbrace{A\left[2\pi f_c + \frac{d\phi(t)}{dt}\right]}_{envelope} \sin\left[2\pi f_c t + \phi(t)\right] \end{aligned} \tag{11.13}$$

The signal is both amplitude- and angle-modulated. If we pass the signal to an envelope detector, we get

$$y(t) = A\,\underbrace{\left[2\pi f_c + \frac{d\phi(t)}{dt}\right]}_{2\pi f_i(t)}$$

$$y(t) = \begin{cases} A\left[2\pi f_c + k_p \dfrac{dm(t)}{dt}\right] & \text{for } PM \\ A\left[2\pi f_c + k_f m(t)\right] & \text{for } FM \end{cases} \tag{11.14}$$

Knowing the values of A, f_c, k_p and k_f, we can compute the desired signal $m(t)$ from $y(t)$. Figure 11.1 shows the circuit for frequency demodulation. The differentiator followed by an envelope detector is called a *frequency discriminator*. For demodulation of PM signals, we simply integrate the output of a frequency discriminator. This yields a signal which is proportional to $m(t)$. Figure 11.2 shows the circuit for phase demodulation.

Figure 11.1 Frequency demodulation using a frequency discriminator.

Figure 11.2 Phase demodulation using a frequency discriminator and an integrator.

In practice, channel noise and other factors may cause A to vary. If A varies, $y(t)$ will vary with A. Hence, it is essential to maintain the amplitude of the input signal to the frequency discriminator. A *hard limiter* is usually used to eliminate any amplitude variations. A hard limiter is a device which limits the output signal to (say) +1 or -1 volt. Figure 11.3 shows the input-output characteristic of a hard limiter.

Figure 11.3 Input-output characteristic of a hard-limiter.

Zero-Crossing Detection.

We have seen that a hard limiter is usually used to eliminate any amplitude fluctuation. The message signal must therefore be contained in the points where the angle-modulated signal crosses the zero voltage level. This produces a means of demodulating an angle-modulated signal. Consider the angle-modulated signal as shown in Figure 11.4 [3].

Figure 11.4 Zero-crossing determination.

Let t_1 and t_2 be two adjacent zero-crossing points, where $t_2 > t_1$. Integrating equation (11.6), we have

$$\int_{t_1}^{t_2} d\theta(t) = \int_{t_1}^{t_2} 2\pi f_i(t)\,dt \tag{11.15}$$

$$\theta(t_2) - \theta(t_1) = \begin{cases} \int_{t_1}^{t_2} [2\pi f_c + k_p \frac{dm(t)}{dt}]dt & \text{for } PM \\ \int_{t_1}^{t_2} [2\pi f_c + k_f m(t)]dt & \text{for } FM \end{cases} \qquad (11.16)$$

but also

$$\theta(t_2) - \theta(t_1) = \pi \qquad (11.17)$$

For $f_c >> B$ (the bandwidth of the message signal), $dm(t)/dt$ for PM signals and $m(t)$ for FM signals change much more slowly than f_c. $dm(t)/dt$ and $m(t)$ may be assumed constant in the interval $t_2 - t_1$. We can write

$$\pi \approx \begin{cases} \underbrace{[2\pi f_c + k_p \frac{dm(t)}{dt}]}_{2\pi f_i(t)}(t_2 - t_1) & \text{for} PM \\ \underbrace{[2\pi f_c + k_f m(t)]}_{2\pi f_i(t)}(t_2 - t_1) & \text{for} FM \end{cases} \qquad (11.18)$$

$$\pi = 2\pi f_i(t)[t_2 - t_1]$$

$$f_i(t) = \frac{1}{2(t_2 - t_1)} \qquad (11.19)$$

where

$$f_i(t) = \begin{cases} f_c + \frac{1}{2\pi} k_p \frac{dm(t)}{dt} & \text{for } PM \\ f_c + \frac{1}{2\pi} k_f m(t) & \text{for} FM \end{cases} \qquad (11.20)$$

Knowing the values of f_c, k_p and k_f, the desired signal $m(t)$ may be found by measuring the spacing between zero crossings in the interval $t_2 - t_1$. A detector utilising this technique is called a *zero-crossing detector*. For demodulation of PM signals, we simply integrate the output of a zero-crossing detector. Again, this yields a signal which is proportional to $m(t)$.

In practice, we consider counting n number of zero-crossings in a time interval T, where

$$\frac{1}{f_c} << T << \frac{1}{B} \qquad (11.21)$$

and B is the bandwidth of the message signal. This is shown in Figure 11.5.

Figure 11.5 Counting intervals.

Then, the number of zero crossings in a time interval T is

$$n = \frac{T}{2\,(t_2 - t_1)}$$

$$\frac{n}{T} = f_i\,(t) \qquad (11.22)$$

References

[1] H. P. Hsu, Analog and Digital Communications, McGraw-Hill, 1993.

[2] L. W. Couch II, Digital and Analog Communication Systems, 5/e, Prentice Hall, 1997.

[3] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw-Hill, 1990.

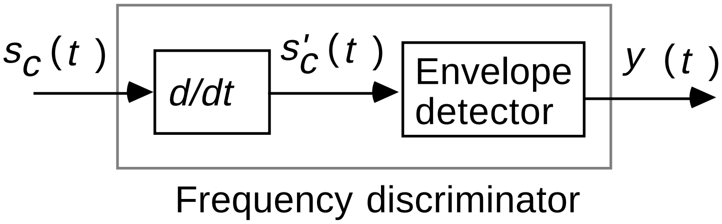


Figure 11.1 Frequency demodulation using a frequency discriminator.

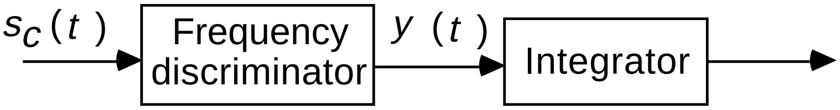


Figure 11.2 Phase demodulation using a frequency discriminator and an integrator.

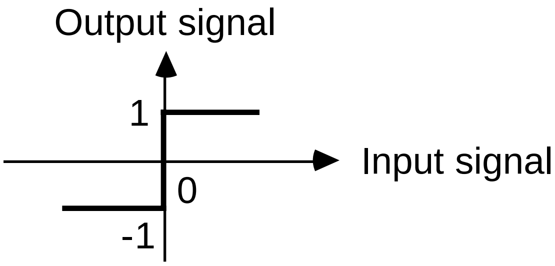
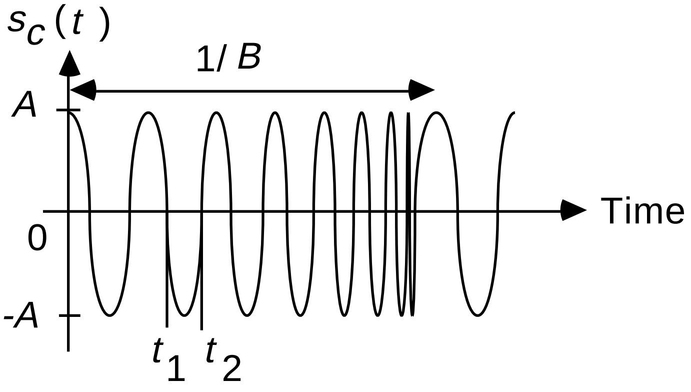


Figure 11.3 Input-output characteristic of a hard limiter.



B - Bandwidth of the message signal

Figure 11.4 Frequency determination.

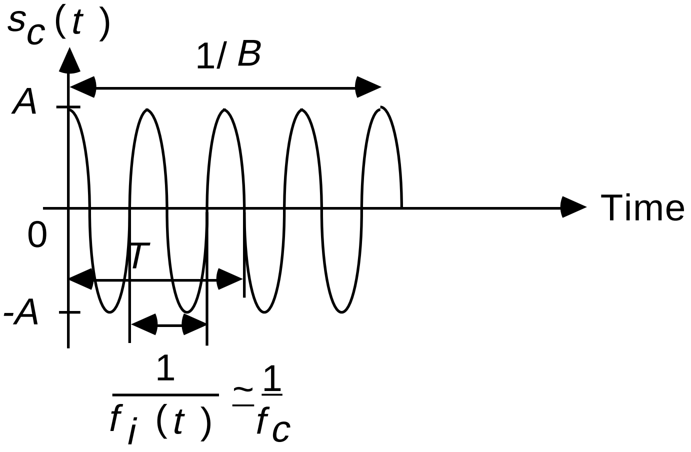


Figure 11.5 Counting intervals.