24. Frequency-Shift Keying (FSK) Modulation and Quadrature Amplitude Modulation (QAM)

Binary Frequency-Shift Keying (BFSK) [1-3]

A *binary frequency-shift keying* (BFSK) signal can be defined by

$$s(t) = \begin{cases} A\cos 2\pi f_0 t, & 0 \le t \le T \\ A\cos 2\pi f_1 t, & elsewhere \end{cases} \qquad (24.1)$$

where A is a constant, f_0 and f_1 are the transmitted frequencies, and T is the bit duration. The signal has a power $P = A^2/2$, so that $A = \sqrt{2P}$. Thus equation (24.1) can be written as

$$\begin{aligned} s(t) &= \begin{cases} \sqrt{2P}\cos 2\pi f_0 t, & 0 \le t \le T \\ \sqrt{2P}\cos 2\pi f_1 t, & elsewhere \end{cases} \\ &= \begin{cases} \sqrt{PT}\sqrt{\frac{2}{T}}\cos 2\pi f_0 t, & 0 \le t \le T \\ \sqrt{PT}\sqrt{\frac{2}{T}}\cos 2\pi f_1 t, & elsewhere \end{cases} \\ &= \begin{cases} \sqrt{E}\sqrt{\frac{2}{T}}\cos 2\pi f_0 t, & 0 \le t \le T \\ \sqrt{E}\sqrt{\frac{2}{T}}\cos 2\pi f_1 t, & elsewhere \end{cases} \end{aligned} \qquad (24.2)$$

where $E = PT$ is the energy contained in a bit duration. For orthogonality, $f_0 = m/T$ and $f_1 = n/T$ for integer $n >$ integer m and $f_1 - f_0$ must be an integer multiple of $1/2T$. We can take $\phi_1(t) = \sqrt{\frac{2}{T}}\cos\ 2\pi f_0 t$ and $\phi_2(t) = \sqrt{\frac{2}{T}}\sin\ 2\pi f_1 t$ as the orthonormal basis functions [3]. The applicable signal constellation diagram of the orthogonal BFSK signal is shown in Figure 24.1.

Figure 24.1 Orthogonal BFSK signal constellation diagram.

Figure 24.2 shows the BFSK signal sequence generated by the binary sequence 0 1 0 1 0 0 1.

Figure 24.2 (a) Binary sequence, (b) BFSK signal, and (c) binary modulating and BASK signals.

It can be seen that phase continuity is maintained at transitions. Further, the BFSK signal is the sum of two BASK signals generated by two modulating signals $m_0(t)$ and $m_1(t)$. Therefore, the Fourier transform of the BFSK signal $s(t)$ is

$$\begin{aligned} S(f) &= \frac{A}{2}\int_{-\infty}^{\infty}[m_0(t)\ e^{j\,2\pi f_0 t}]\ e^{-j2\pi ft}\,dt + \\ &\quad \frac{A}{2}\int_{-\infty}^{\infty}[m_0(t)\ e^{-j\,2\pi f_0 t}]\ e^{-j2\pi ft}\,dt + \\ &\quad \frac{A}{2}\int_{-\infty}^{\infty}[m_1(t)\ e^{j\,2\pi f_1 t}]\ e^{-j2\pi ft}\,dt + \\ &\quad \frac{A}{2}\int_{-\infty}^{\infty}[m_1(t)\ e^{-j\,2\pi f_1 t}]\ e^{-j2\pi ft}\,dt \\ &= \frac{A}{2}M_0(f - f_0) + \frac{A}{2}M_0(f + f_0) + \\ &\quad \frac{A}{2}M_1(f - f_1) + \frac{A}{2}M_1(f + f_1) \end{aligned} \tag{24.3}$$

Figure 24.3 shows the amplitude spectrum of the BFSK signal when $m_0(t)$ and $m_1(t)$ are periodic pulse trains.

Figure 24.3 (a) Modulating signals, (b) Spectrum of (a), and (c) spectrum of BFSK signal (positive frequencies only).

An alternative representation of the BFSK signal consists of letting $f_0 = f_c - \Delta f$ and $f_1 = f_c + \Delta f$. Then

$$f_1 - f_0 = 2\Delta f \tag{24.4}$$

and

$$s(t) = A\cos 2\pi(f_c + \Delta f)t \tag{24.5}$$

where f_c is the carrier frequency, $\Delta f = \beta\, B$ is the *frequency deviation*, β is the *modulation index*, and $B = 1/T$ is the bandwidth of the modulating signal. When $\Delta f >> 1/T$, we have a wideband BFSK signal. The bandwidth is approximately equal to $2\,\Delta f$. When $\Delta f << 1/T$, we have a narrowband BFSK signal. The bandwidth is approximately equal to $2B$.

Figure 24.4 shows the modulator and coherent demodulator for BFSK signals [2].

Figure 24.4 (a) BFSK modulator and (b) coherent demodulator.

M-ary Frequency-Shift Keying (*M*-FSK) [2-4]

An *M-ary frequency-shift keying* (*M*-FSK) signal can be defined by

$$s(t) = \begin{cases} A\cos(2\pi f_i t + \theta'), & 0 \le t \le T \\ 0, & elsewhere \end{cases} \qquad (24.6)$$

for $i = 0, 1, ..., M - 1$. Here, A is a constant, f_i is the transmitted frequency, θ' is the initial phase angle, and T is the symbol duration. It has a power $P = A^2/2$, so that $A = \sqrt{2P}$. Thus equation (24.6) can be written as

$$\begin{aligned} s(t) &= \sqrt{2P}\ \cos(2\pi f_i t + \theta'), && 0 \le t \le T \\ &= \sqrt{PT}\sqrt{\frac{2}{T}}\cos(2\pi f_i t + \theta'), && 0 \le t \le T \\ &= \sqrt{E}\sqrt{\frac{2}{T}}\cos(2\pi f_i t + \theta'), && 0 \le t \le T \end{aligned} \qquad (24.7)$$

where $E = PT$ is the energy of $s(t)$ contained in a symbol duration for $i = 0, 1, ..., M - 1$. For convenience, the arbitrary phase angle θ' is taken to be zero. If we choose $f_0 = k/T$, $f_1 = (k + 2)/T$, $f_3 = (k + 4)/T$, ..., $k > 0$, we can take $\phi_1(t) = \sqrt{\frac{2}{T}}\cos\ 2\pi f_0 t$, $\phi_2(t) = \sqrt{\frac{2}{T}}\sin\ 2\pi f_1 t$, ... as the orthonormal basis functions [3]. Figure 24.5 shows the signal constellation diagram of an orthogonal 3-FSK signal.

Figure 24.5 Orthogonal 3-FSK signal constellation diagram.

Figure 24.6 shows the 4-FSK signal generated by the binary sequence 00 01 10 11.

Figure 24.6 4-FSK modulation: (a) binary signal and (b) 4-FSK signal.

Figure 24.7 shows the modulator and coherent demodulator for *M*-FSK signals [4]. The mapping table simply maps the detected index *i* onto a binary vector.

Figure 24.7 (a) *M*-FSK modulator and (b) coherent demodulator.

M-ary Quadrature Amplitude Modulation (*M*-QAM)

An *M-ary quadrature amplitude modulation* (*M*-QAM) signal can be defined by

$$s(t) = \begin{cases} A_i \cos(2\pi f_c t + \theta_i), & 0 \le t \le T \\ 0, & elsewhere \end{cases} \quad (24.8)$$

$$= \begin{cases} A_i \cos\theta_i \cos 2\pi f_c t - A_i \sin\theta_i \sin 2\pi f_c t, & 0 \le t \le T \\ 0, & elsewhere \end{cases} \quad (24.9)$$

for i = 0, 1, ..., M - 1. Here, A_i is the amplitude, f_c is the carrier frequency, θ_i is the phase angle, and T is the symbol duration. It has a power $P_i = A_i^2/2$, so that $A_i = \sqrt{2P_i}$. Thus equation (24.9) can be written as

$$\begin{aligned} s(t) &= \sqrt{P_i T}\cos\theta_i \sqrt{\frac{2}{T}}\cos 2\pi f_c t - \sqrt{P_i T}\sin\theta_i \sqrt{\frac{2}{T}}\sin 2\pi f_c t \\ &= \sqrt{E_i}\cos\theta_i \sqrt{\frac{2}{T}}\cos 2\pi f_c t - \sqrt{E_i}\sin\theta_i \sqrt{\frac{2}{T}}\sin 2\pi f_c t \end{aligned} \quad (24.10)$$

where $E_i = P_i T$ is the energy of $s(t)$ contained in a symbol duration for i = 0, 1, ..., M - 1. If we take $\phi_1(t) = \sqrt{\frac{2}{T}}\cos\ 2\pi f_c t$ and $\phi_2(t) = -\sqrt{\frac{2}{T}}\sin\ 2\pi f_c t$ as the orthonormal basis functions, the applicable signal constellation diagrams of the 16-QAM and 4-QAM signals are shown in Figure 24.8.

Figure 24.8 (a) 16-QAM and (b) 4-QAM signal constellation diagrams.

Figure 24.9 shows the modulator and a possible implementation of the coherent demodulator for *M*-QAM signals.

Figure 24.9 (a) *M*-QAM modulator and (b) coherent demodulator.

References

[1] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw Hill, 1990.

[2] P. Z. Peebles, Jr., Digital Communication Systems, Prentice Hall, 1987.

[3] H. Taub and D. L. Schilling, Principles of Communication Systems, 2/e, McGraw Hill, 1986

[4] F. Xiong, Digital Modulation Techniques, Artech House, 2000.

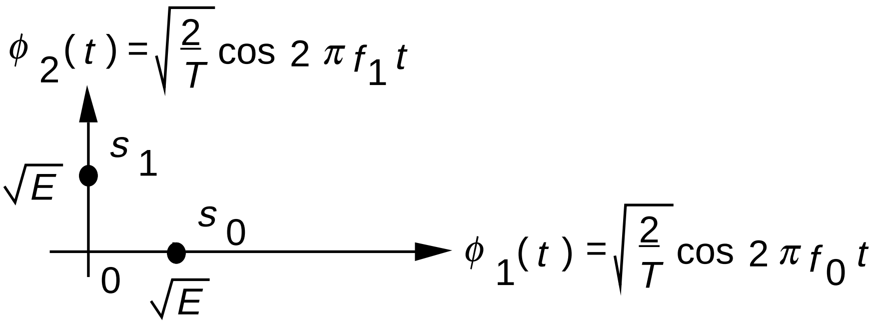


Figure 24.1 Orthogonal BFSK signal constellation diagram.

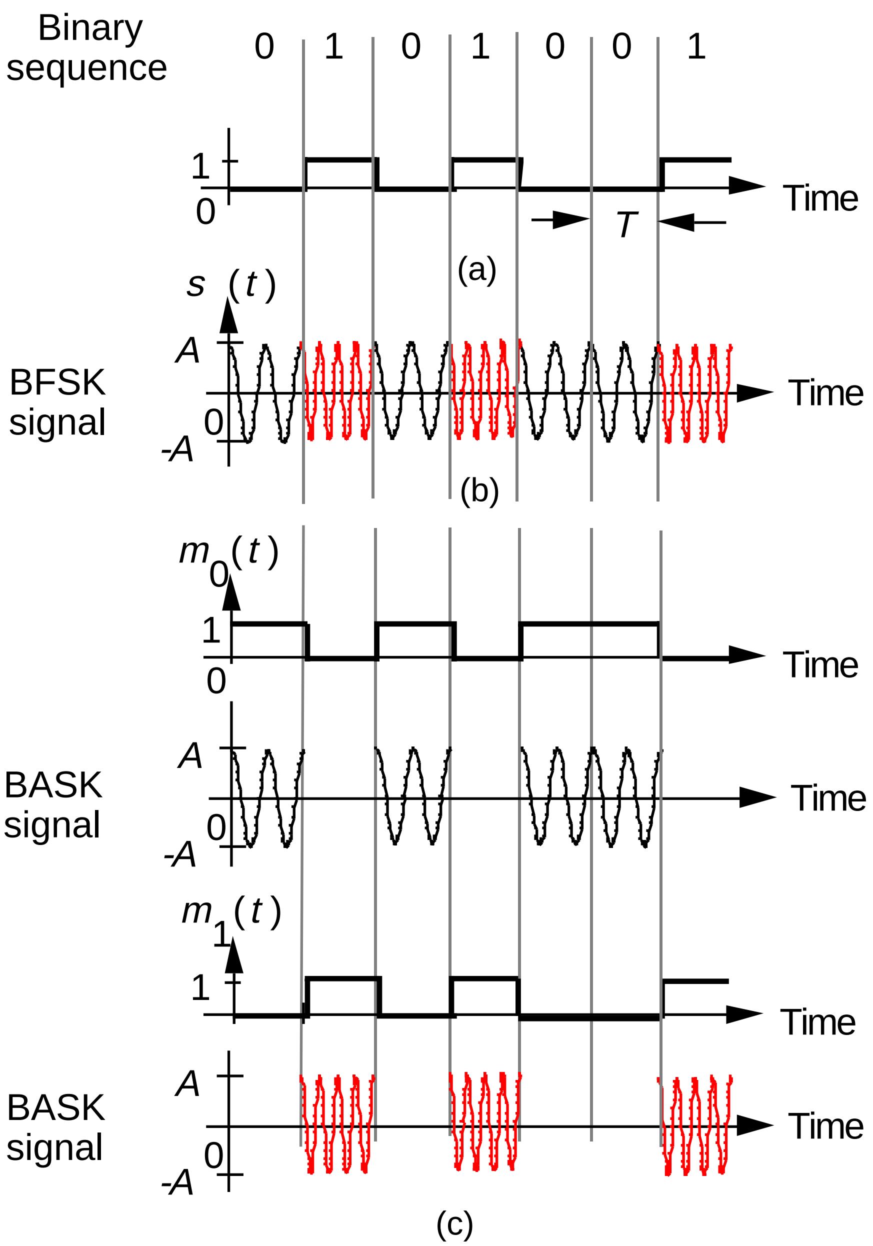


Figure 24.2 (a) Binary sequence, (b) BFSK signal, and (c) binary modulating and BASK signals.

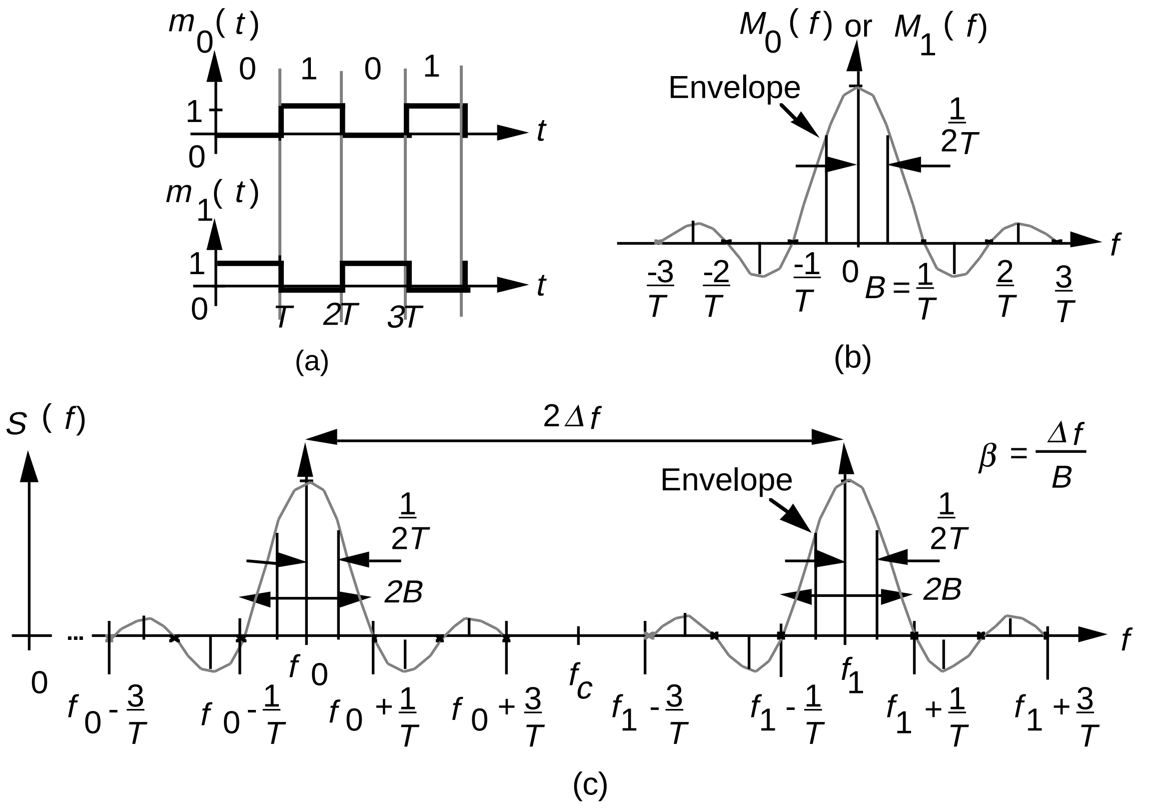


Figure 24.3 (a) Modulating signals, (b) Spectrum of (a), and (c) spectrum of BFSK signal (positive frequencies only).

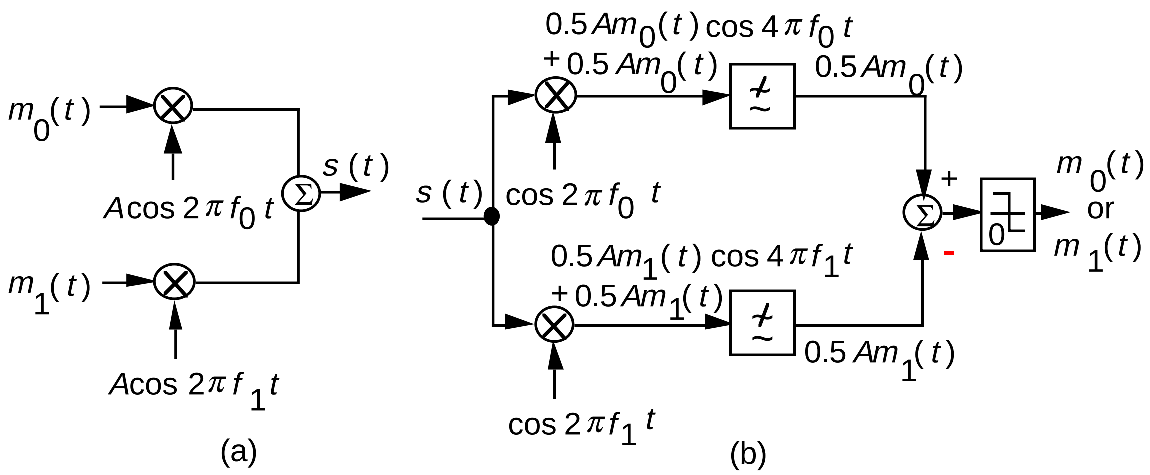


Figure 24.4 (a) BFSK modulator and (b) coherent demodulator.

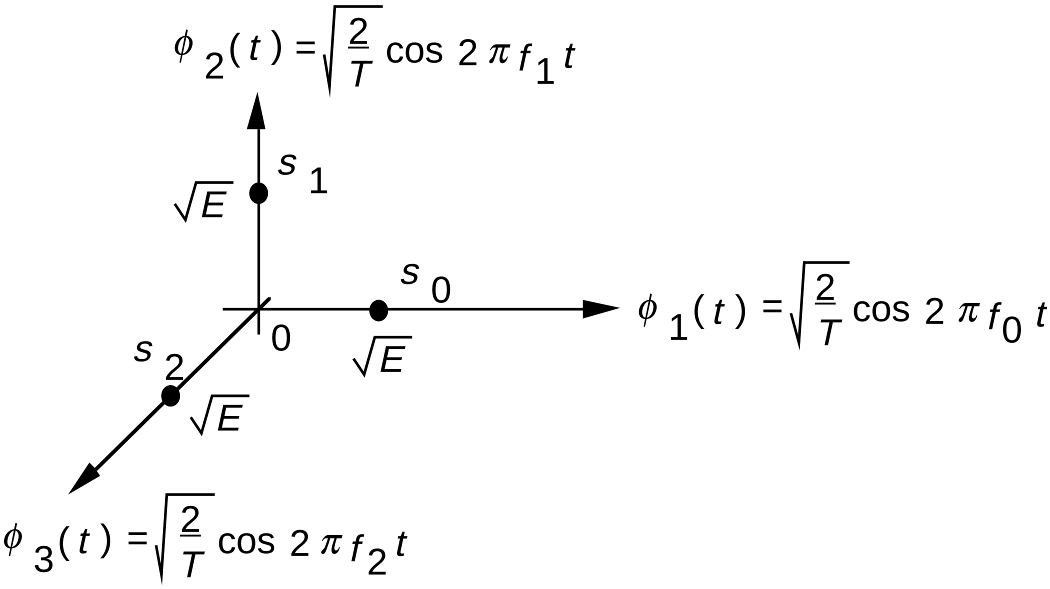


Figure 24.5 Orthogonal 3-FSK signal constellation diagram.

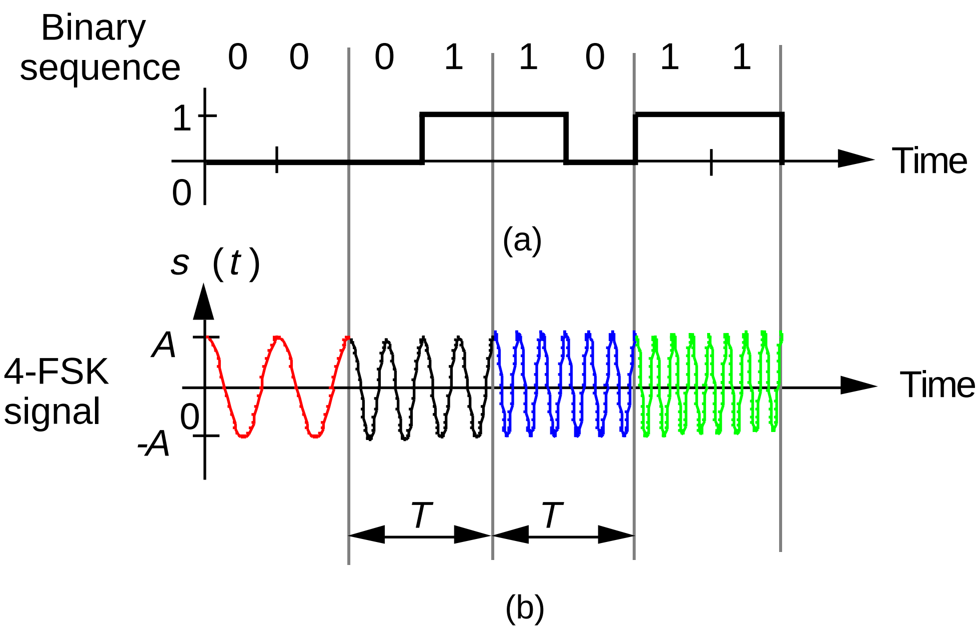
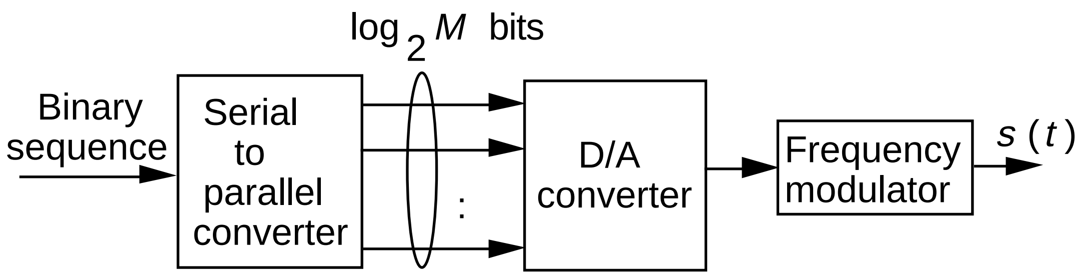
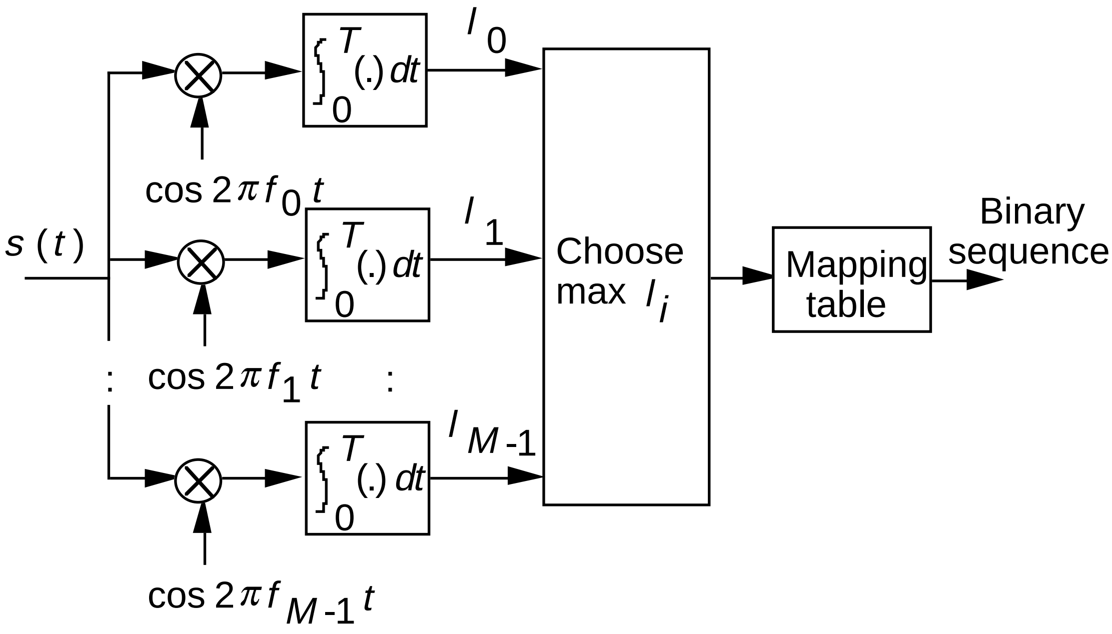


Figure 24.6 4-FSK modulation: (a) binary signal and (b) 4-FSK signal.



(a)



(b)

Figure 24.7 (a) *M*-FSK modulator and (b) coherent demodulator.

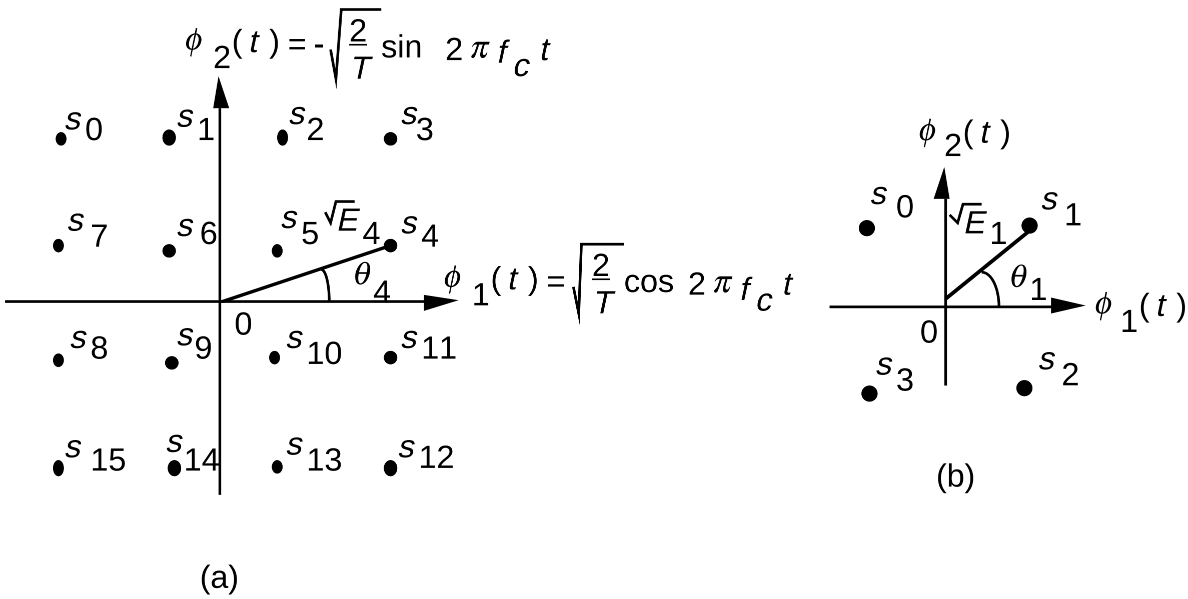


Figure 24.8 (a) 16-QAM and (b) 4-QAM signal constellation diagrams.

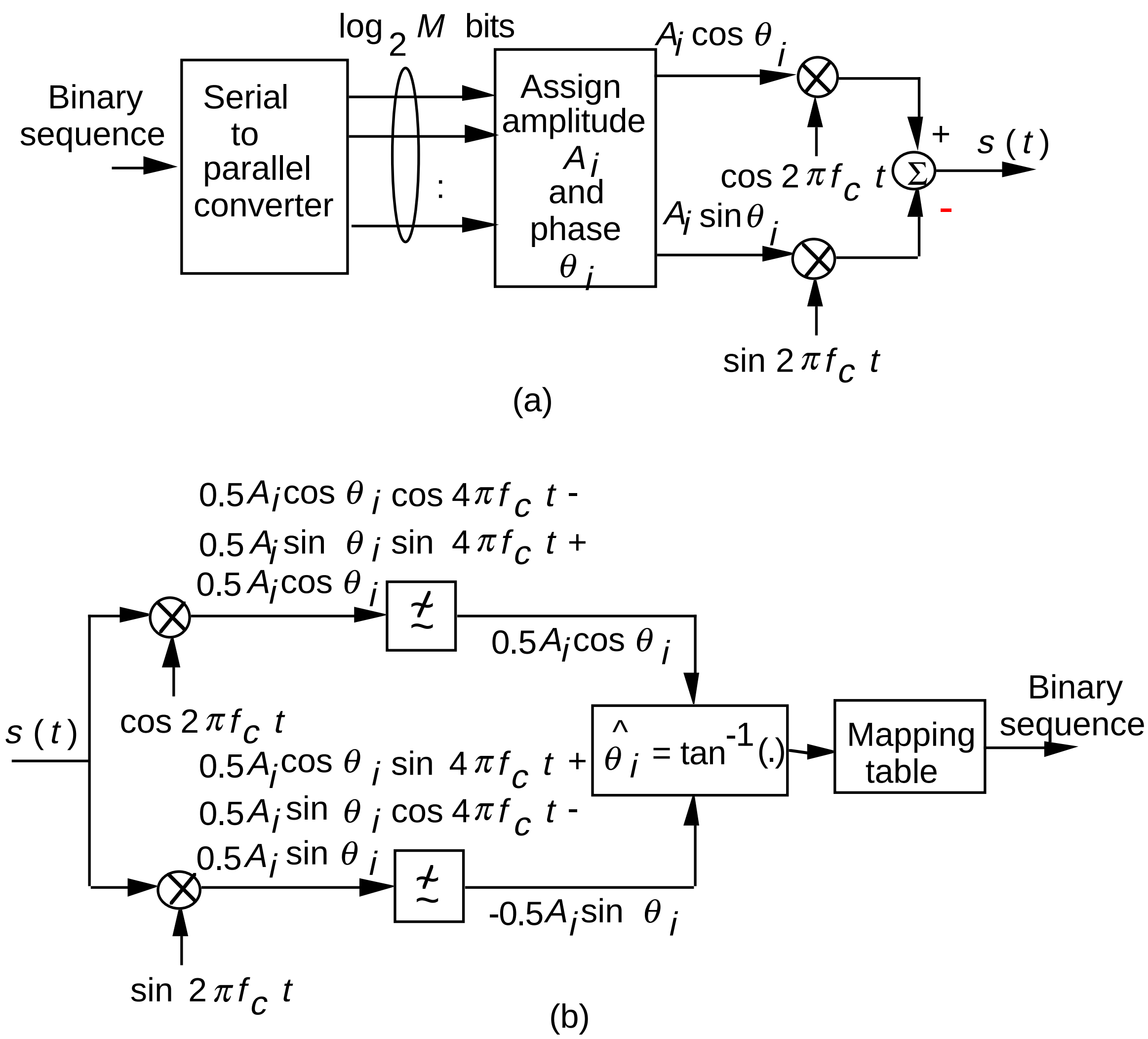


Figure 24.9 (a) *M*-QAM modulator and (b) coherent demodulator.