23. Phase-Shift Keying (PSK) Modulation

Binary Phase-Shift Keying (BPSK) [1]

A *binary phase-shift keying* (BPSK) signal can be defined by

$$s(t) = A\, m(t) \cos 2\pi f_c t, \qquad 0 \le t \le T \tag{23.1}$$

where A is a constant, $m(t) = +1$ or -1, f_c is the carrier frequency, and T is the bit duration. The signal has a power $P = A^2/2$, so that $A = \sqrt{2P}$. Thus equation (23.1) can be written as

$$\begin{aligned} s(t) &= \pm\sqrt{2P}\cos 2\pi f_c t \\ &= \pm\sqrt{PT}\sqrt{\frac{2}{T}}\cos 2\pi f_c t \\ &= \pm\sqrt{E}\sqrt{\frac{2}{T}}\cos 2\pi f_c t \end{aligned} \tag{23.2}$$

where $E = PT$ is the energy contained in a bit duration. If we take $\phi_1(t) = \sqrt{\frac{2}{T}}\cos 2\pi f_c t$ as the orthonormal basis function, the applicable signal constellation diagram of the BPSK signals is shown in Figure 23.1.

Figure 23.1 BPSK signal constellation diagram.

Figure 23.2 shows the BPSK signal sequence generated by the binary sequence 0 1 0 1 0 0 1.

Figure 23.2 (a) Binary modulating signal, and (b) BPSK signal.

Figure 23.3 shows the amplitude spectrum of the BPSK signals when $m(t)$ is a periodic pulse train. The spectrum of the BPSK signals is that of a double-sideband suppressed-carrier signal.

Figure 23.3 (a) Modulating signal, (b) Spectrum of (a), and (c) spectrum of BPSK signals.

Since we define the bandwidth as the range occupied by the baseband signal $m(t)$ from 0 Hz to the first zero-crossing point, we have B Hz of bandwidth for the baseband signal and $2B$ Hz for the BPSK signal. Figure 23.4 shows the modulator and a possible implementation of the coherent demodulator for BPSK signals.

Figure 23.4 (a) BPSK modulator, and (b) coherent demodulator.

M-ary Phase-Shift Keying (*M*-PSK) [2-4]

An *M-ary phase-shift keying* (*M*-PSK) signal can be defined by

$$s(t) = \begin{cases} A\cos(2\pi f_c t + \theta_i + \theta'), & 0 \le t \le T \\ 0, & elsewhere \end{cases} \tag{23.3}$$

where

$$\theta_i = \frac{2\pi}{M} i \tag{23.4}$$

for i = 0, 1, ..., M - 1. Here, A is a constant, f_c is the carrier frequency, θ' is the initial phase angle, and T is the symbol duration. By expanding equation (23.3), we have

$$s(t) = A\cos\theta_i \cos(2\pi f_c t+\theta') - A\sin\theta_i \sin(2\pi f_c t+\theta') \tag{23.5}$$

The signal has a power $P = A^2/2$, so that $A = \sqrt{2P}$. Thus equation (23.5) can be written as

$$\begin{aligned} s(t) &= \sqrt{PT}\cos\theta_i \sqrt{\frac{2}{T}}\cos(2\pi f_c t+\theta') - \sqrt{PT}\sin\theta_i \sqrt{\frac{2}{T}}\sin(2\pi f_c t+\theta') \\ &= \sqrt{E}\cos\theta_i \sqrt{\frac{2}{T}}\cos(2\pi f_c t+\theta') - \sqrt{E}\sin\theta_i \sqrt{\frac{2}{T}}\sin(2\pi f_c t+\theta') \end{aligned} \tag{23.6}$$

where $E = PT$ is the energy of $s(t)$ contained in a symbol duration for i = 0, 1, ..., M - 1. For convenience, the arbitrary phase angle θ' is taken to be zero. If we take $\phi_1(t) = \sqrt{\frac{2}{T}}\cos 2\pi f_c t$ and $\phi_2(t) = -\sqrt{\frac{2}{T}}\sin 2\pi f_c t$ as the orthonormal basis functions, the applicable signal constellation diagrams of the M-PSK and 4-PSK signals are shown in Figure 23.5. All signal points lie on a circle of radius $\sqrt{E}$.

Figure 23.5 (a) *M*-PSK and (b) 4-PSK signal constellation diagrams.

Figure 23.6 shows the 4-PSK signal sequence generated by the binary sequence 00 01 10 11.

Figure 23.6 4-PSK modulation: (a) binary sequence and (b) 4-PSK signal.

Figure 23.7 shows the modulator and a possible implementation of the coherent demodulator for *M*-PSK signals [3, 4].

Figure 23.7 (a) *M*-PSK modulator, and (b) coherent demodulator.

We may use the mapping table shown in Table 23.1 for the phase-to-binary-vector conversion.

θ_i	Natural binary vector
0	0 0 ... 0 0
$\frac{2\pi}{M}$	0 0 ... 0 1
$2(\frac{2\pi}{M})$	0 0 ... 1 0
:	:
$(M\text{-}1)\frac{2\pi}{M}$	1 1 ... 1 1

Table 23.1 A possible mapping table for *M*-PSK coherent demodulation.

References

[1] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw Hill, 1990.

[2] P. Z. Peebles, Jr., Digital Communication Systems, Prentice Hall, 1987.

[3] J. D. Gibson, Principles of Digital and Analog Communications, MacMillan, 1990.

[4] B. Sklar, Digital Communications: Fundamentals and Applications, Prentice Hall, 1988.

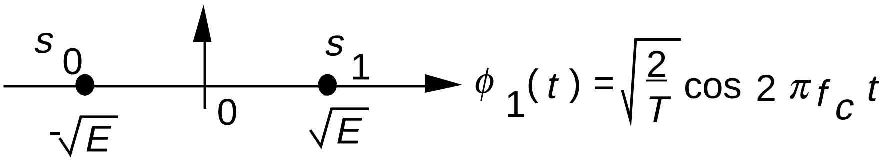

Figure 23.1 BPSK signal constellation diagram.

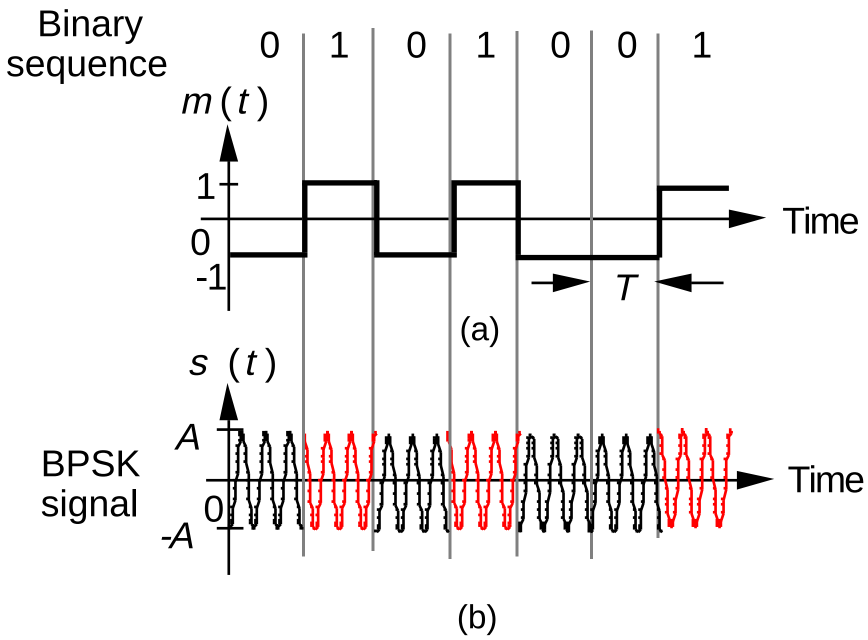

Figure 23.2 (a) Binary modulating signal, and (b) BPSK signal.

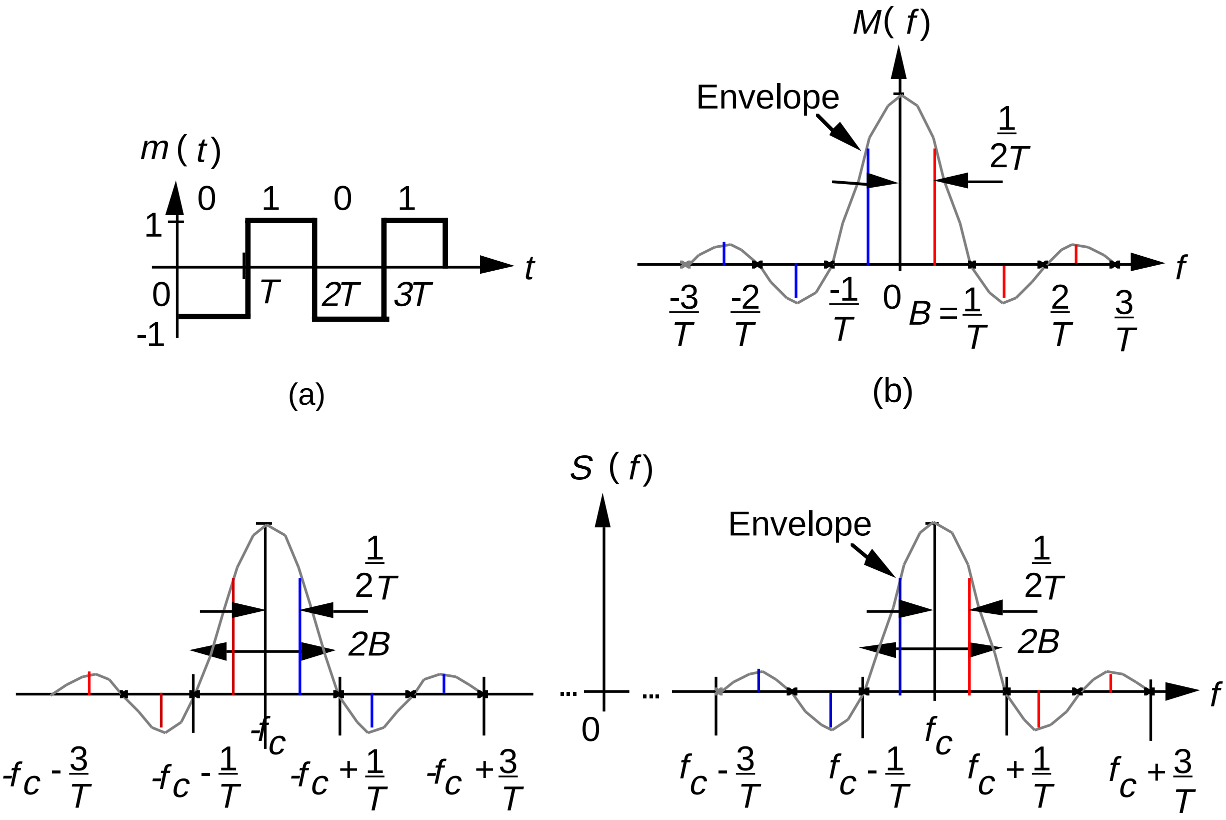

Figure 23.3 (a) Modulating signal, (b) Spectrum of (a), and (c) spectrum of BPSK signals.

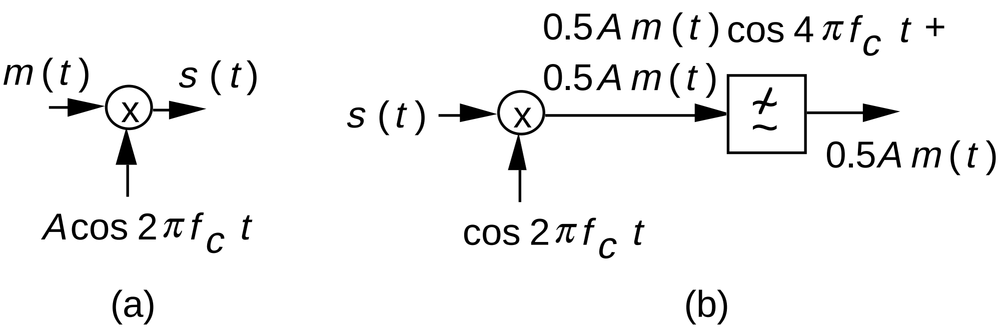

Figure 23.4 (a) BPSK modulator, and (b) coherent demodulator.

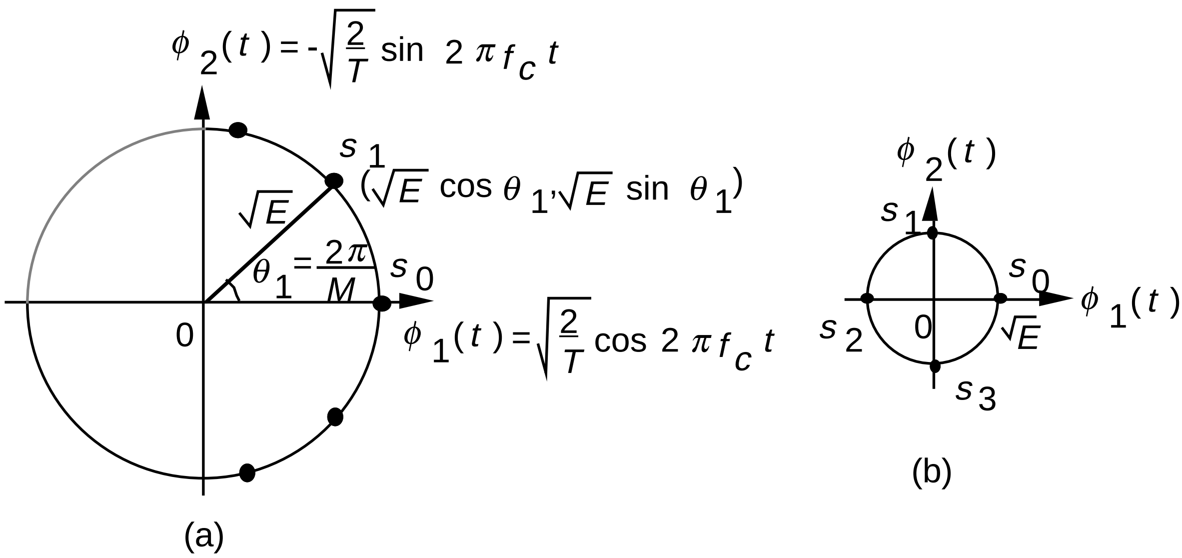

Figure 23.5 (a) *M*-PSK and (b) 4-PSK signal constellation diagrams.

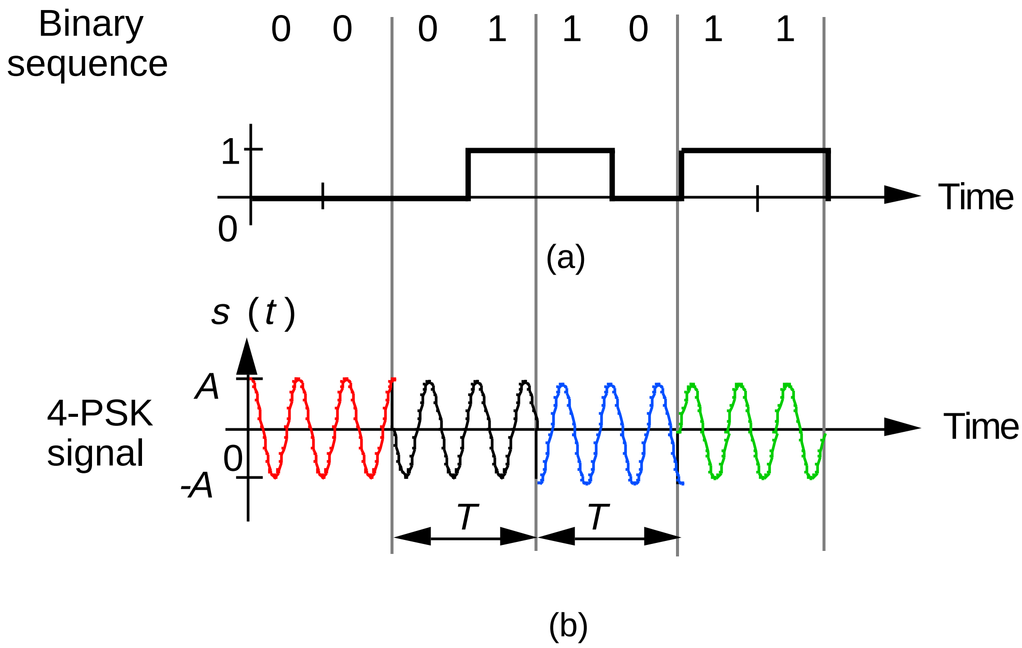

Figure 23.6 4-PSK modulation: (a) binary sequence and (b) 4-PSK signal.

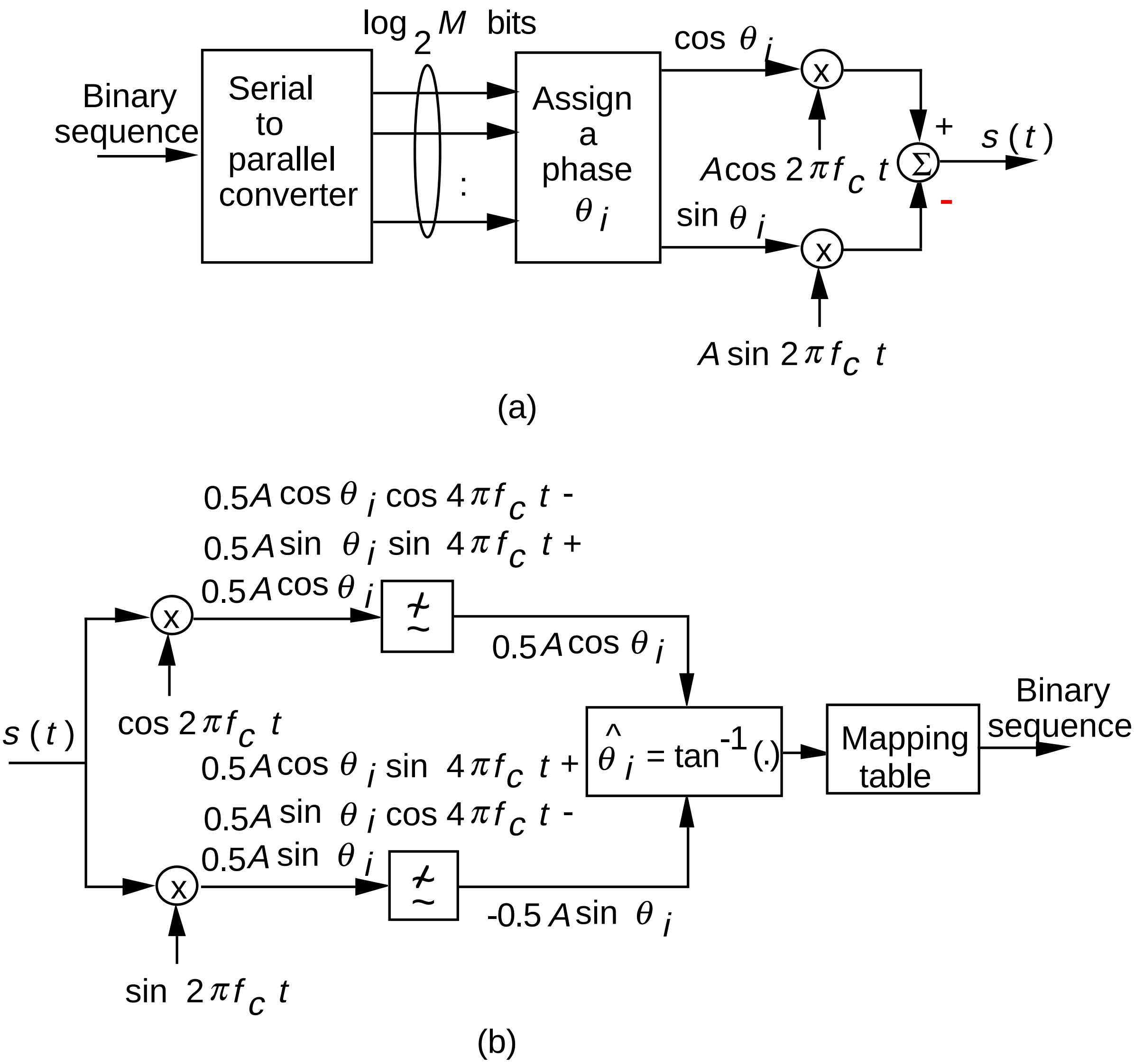

Figure 23.7 (a) *M*-PSK modulator, and (b) coherent demodulator.