# 14. Pulse Code Modulation

We have seen that sampling a bandlimited signal at or above the Nyquist sampling rate does not destroy any information content and fully characterises the bandlimited signal. A system transmitting these sampled values of the bandlimited signal is called a *sampled-data* or *pulse modulation system*. In modern communication systems, these sampled signals are often quantised and coded before transmission. We have *pulse code modulation (PCM)*.

Pulse code modulation is very popular because of the many advantages it offers. These include:

1. Inexpensive digital circuitry may be used in the system.
2. All-digital transmission. PCM signals derived from analogue signals may be time-division multiplexed with data from digital computers and transmitted over a common high-speed channel.
3. Further digital signal processing such as *encryption* is possible.
4. Errors may be minimised by appropriate coding of the signals.
5. Signals may be regularly reshaped or regenerated using *repeaters* at appropriate intervals.

Figure 14.1 shows a single-channel PCM system.

**Figure 14.1** A single-channel PCM transmission system.

An analogue message $m(t)$ is first sampled at or above the Nyquist sampling rate. These sampled signals are then converted into a finite number of discrete amplitude levels. The conversion process is called *quantisation*. Figure 14.2 shows how an analogue message is converted into 8 amplitude levels with equal spacing by an 8-level quantiser.

**Figure 14.2** Message and quantised signal.

Quantisation obviously reduces the degree of accuracy of representation of the sampled signal and introduces some error in the reproduction of the signal at the receiver. Error introduced by the quantiser is called *quantisation error* or *quantisation noise*. To reduce the quantisation error, we simply increase the total number of amplitude levels (decreasing the spacing between adjacent levels). What is the minimum number of quantisation levels for speech? 8 ~ 16 levels are sufficient. In practical digital telephone systems, $256 = 2^8$ levels are used to keep the quantisation error to a tolerable level. $65{,}536 = 2^{16}$ levels are used for the CD digital system.

If the quantised samples are transmitted directly over a channel, we have a *quantised PAM system*. If, instead, we code each quantised sample into a block of digits for transmission, , we have a *PCM system*. The decimal-to-binary conversion can be done in various ways. Table 15.1 shows two possible coding rules (binary and gray coding) for converting a 16-level sample into 4 binary digits.

| Digit | Binary Code $[b_1\ b_2\ b_3\ b_4]$ | Gray Code $[g_1\ g_2\ g_3\ g_4]$ |
|---|---|---|
| 0 | 0 0 0 0 | 0 0 0 0 |
| 1 | 0 0 0 1 | 0 0 0 1 |
| 2 | 0 0 1 0 | 0 0 1 1 |
| 3 | 0 0 1 1 | 0 0 1 0 |
| 4 | 0 1 0 0 | 0 1 1 0 |
| 5 | 0 1 0 1 | 0 1 1 1 |
| 6 | 0 1 1 0 | 0 1 0 1 |
| 7 | 0 1 1 1 | 0 1 0 0 |
| 8 | 1 0 0 0 | 1 1 0 0 |
| 9 | 1 0 0 1 | 1 1 0 1 |
| 10 | 1 0 1 0 | 1 1 1 1 |
| 11 | 1 0 1 1 | 1 1 1 0 |
| 12 | 1 1 0 0 | 1 0 1 0 |
| 13 | 1 1 0 1 | 1 0 1 1 |
| 14 | 1 1 1 0 | 1 0 0 1 |
| 15 | 1 1 1 1 | 1 0 0 0 |

**Table 14.1** Decimal-to-binary conversion.

The elements $b_k$ and $g_k$ are related to each other by the following equations:

$$g_1 = b_1,$$
$$g_k = b_k \oplus b_{k-1},\ k \geq 2 \tag{14.1}$$

$$b_1 = g_1,$$
$$b_k = g_k \oplus b_{k-1},\ k \geq 2 \tag{14.2}$$

where $\oplus$ represents the modulo-2 operation. It can be seen from Table 14.1 that, in changing from one decimal digit to an adjacent digit, the binary code may change by more than one binary digit. This makes the binary code highly susceptible to error in recording the analogue-to-digital conversion. One would prefer a code in which only one binary digit at a time changed as the corresponding input digit changed by one level. Gray code has the above property and is the preferred coding method. Figure 14.3 shows 3 quantised samples and their corresponding coded bit sequences.

**Figure 14.3** Binary and Gray coding of samples.

In Figure 14.4, we show a complete 10-channel PCM system and its associated signal shapes at various transmitting points. Clearly, the bandwidth required at the output of the binary encoder is three times the bandwidth required at the input and the output of the quantiser. Thus, a binary PCM system requires more transmission bandwidth than the PAM and the quantised PAM systems.

**Figure 14.4** Ten-channel PCM system. (a) Transmitter. (b) Receiver. (c) Signal shapes.

### Bandwidth Reduction Technique

Binary coding is just one special case of a coding method in a PCM system. In general, we can code a quantised sample into a group of $m$ pulses every $T$ seconds, each pulse with a duration of $\tau = T/m$ seconds and $n$ possible amplitude levels. Clearly, the total number of amplitude levels that a quantised signal can have is $M = n^m$. The ability to choose $n$ and $m$ gives us some freedom to reduce the transmission bandwidth. Figure 14.5 shows the bandwidth reduction effects when we vary $n$ and $m$. If $n$ is fixed, we can reduce the transmission bandwidth by reducing the value of $m$. This is shown in Figure 14.5 (a). If $M$ is fixed, we can reduce the transmission bandwidth by increasing the value of $n$ and reducing the value of $m$. This is shown in Figure 14.5 (b). The collapsing of successive pulses onto one much wider pulse reduces the transmission bandwidth. However, there is one major drawback for the fixed $M$ case. If the spacing between adjacent levels is fixed, the required peak power goes up as $n$ increases. On the other hand, if the peak power or amplitude swing is fixed, adjacent levels get closer to each other. This makes easier for noise to obscure adjacent levels. Not a very good bandwidth reduction technique! The technique is only useful for very-low-noise environments.

$n = 2$ is the most noise-immune choice. As we are only dealing with on-off signalling, the exact magnitude is not important. Reshaping of signals by repeaters facilitates the signal decision process at the receiver.

**Figure 14.5** Bandwidth reduction technique. (a) Fixed $n$, (b) Fixed $M$.

## References

[1] H. P. Hsu, Analog and Digital Communications, McGraw-Hill, 1993.

[2] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw-Hill, 1990.

[3] L. W. Couch II, Analog and Digital Communication Systems, 6/e, Prentice Hall, 2001.

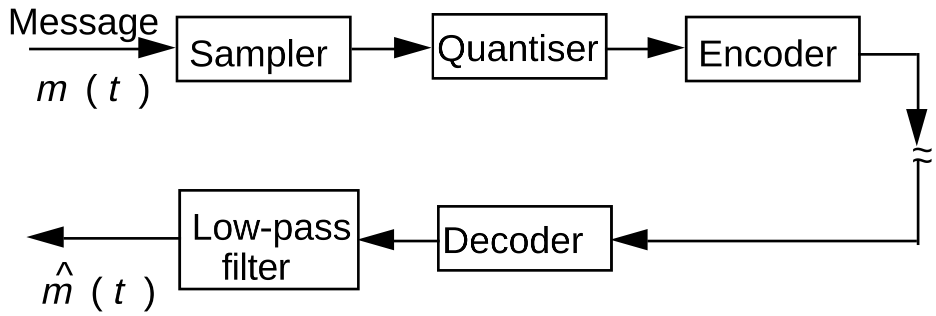


**Figure 14.1** A single-channel PCM transmission system.

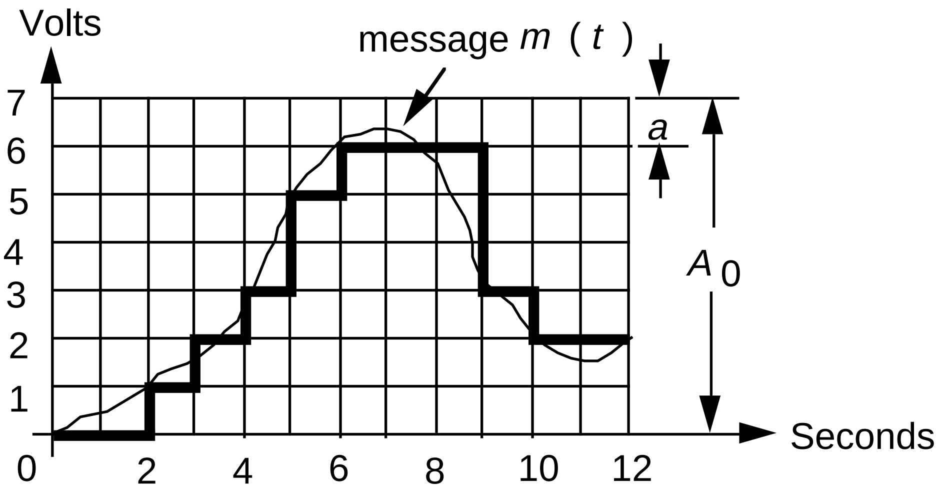


**Figure 14.2** Message and quantised signal.

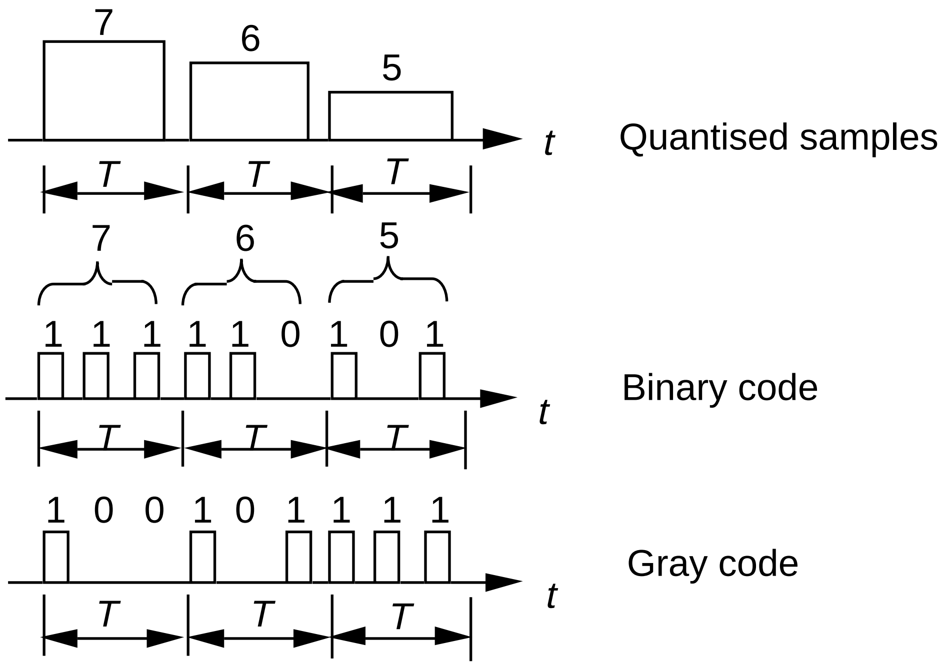


**Figure 14.3** Binary and Gray coding of samples.

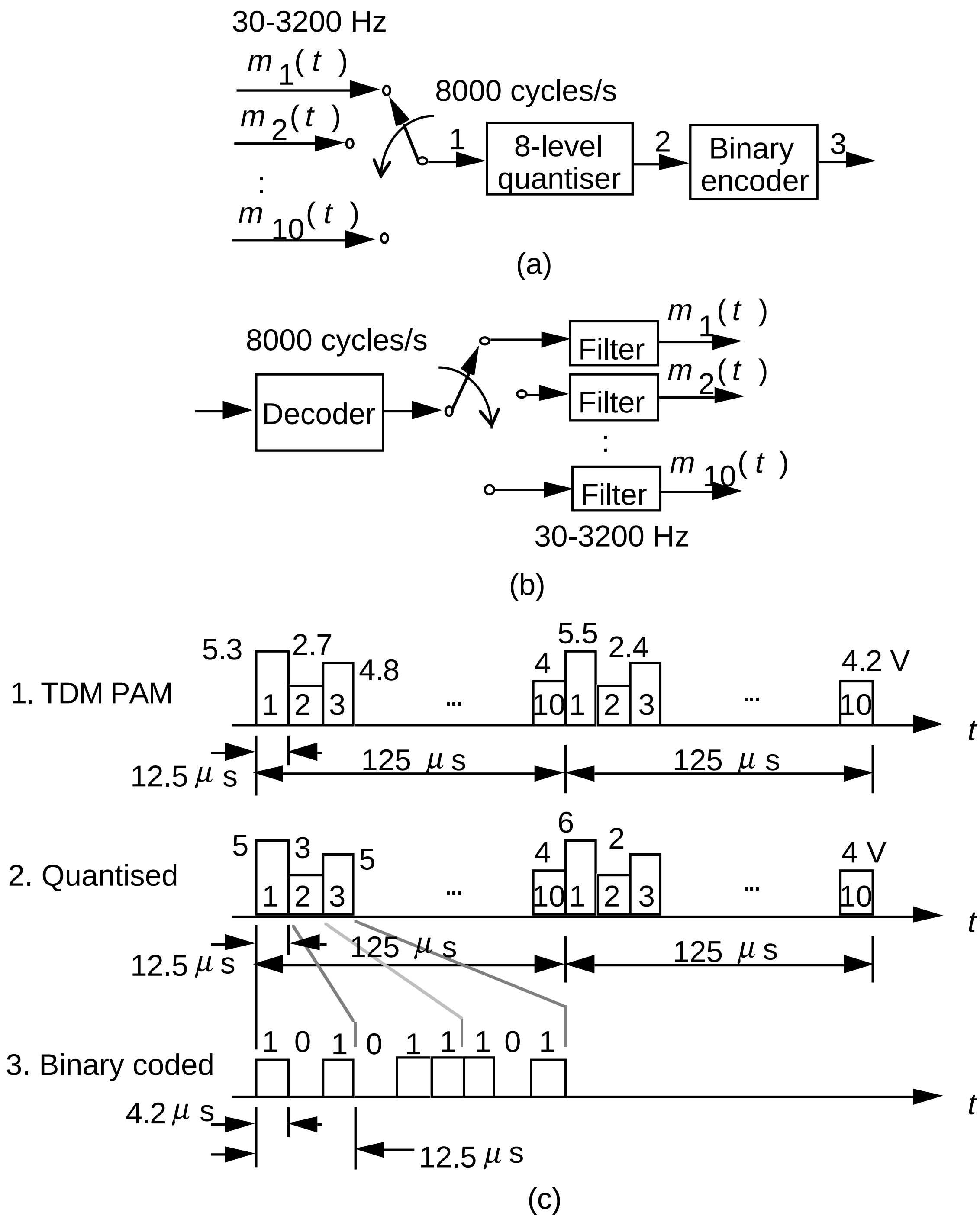


**Figure 14.4** Ten-channel PCM system. (a) Transmitter. (b) Receiver. (c) Signal shapes.

$M = n^m$ Total no. of amplitude levels
$m = T/\tau$ - No. of pulses per sample
$n$ - No. of possible amplitude levels per pulse

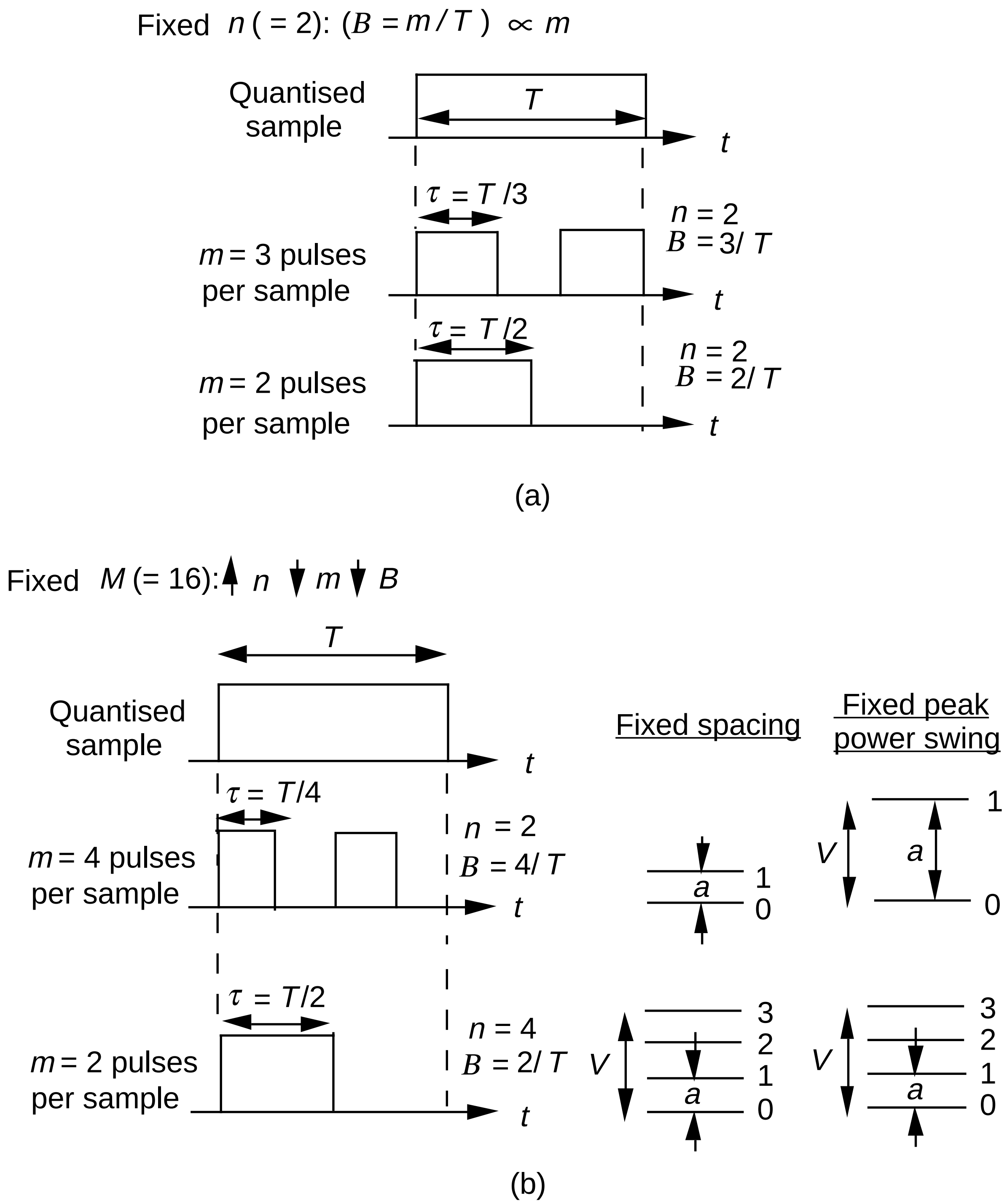


**Figure 14.5** Bandwidth reduction technique. (a) Fixed *n*, (b) Fixed *M*.