19. Waveform Shaping and Eye Patterns

Waveform Shaping for Eliminating Intersymbol Interference

So far, we have simply used rectangular pulses to represent digital signals. In actual practice, these pulses will modulate a carrier signal for transmission over long distances. For most baseband and passband communication systems, our goal is to reduce the required transmission bandwidth as much as possible. What would happen if we operated a communication system beyond the bandwidth of the transmission medium? The result is spreading the pulses in time. Figure 19.1 shows the pulse-spreading effect in a baseband transmission system.

Figure 19.1 Intersymbol interference in a digital baseband transmission system.

If we detect the signals at the sampling instants, overlapping from adjacent signals may result in erroneous detection. This phenomenon of pulse overlap is called *intersymbol interference (ISI)*. This interference can be reduced if we increase the available *channel bandwidth*. Instead, we can shape the pulse at the transmitter to minimise or eliminate this interference effect rather than expanding the channel bandwidth. One obvious choice is to use one that is maximum at the sampling instant, yet goes through zero at all adjacent sampling instants. Figure 19.2 shows a possible pulse waveform.

Figure 19.2 Pulse providing zero intersymbol interference.

The pulse goes through zero at multiples of $T = 1/2B$ seconds. By sampling at multiples of $1/2B$ seconds, pulses of the same shape that are spaced $1/2B$ seconds apart will not interfere with each other. This is shown in Figure 19.3.

Figure 19.3 Transmission of pulses without intersymbol interference.

A maximum of $2B$ pulses per second may be transmitted over a channel bandwidth of B Hz without ISI. This is the *Nyquist rate for no ISI*. However, this particular pulse shape is difficult to implement. This is due to the fact that:

1. $H(f)$ is an ideal brickwall filter.

2. If the sampling instant drifts from the desired sampling instant, the tails of all adjacent pulses may add up as a dominant component and cause a detection error.

What we need is a pulse with a shape that minimises this tailing effect. The raised-cosine filter has the above property. This is shown in Figure 19.4.

Figure 19.4 Raised-cosine filter spectrum.

The transfer function of a raised-cosine filter is given by

$$H(f) = \begin{cases} \frac{1}{2}\left(1 + \cos\frac{\pi f}{2 f_c}\right), & |f| \le 2 f_c \\ 0, & \textit{elsewhere} \end{cases} \qquad (19.1)$$

and the impulse response of the raised-cosine filter is given by

$$h(t) = 2 f_c \frac{\sin 2\pi f_c t}{2\pi f_c t} \frac{\cos 2\pi f_c t}{1 - (4 f_c t)^2} \qquad (19.2)$$

The term $\frac{\sin 2\pi f_c t}{2\pi f_c t}$ ensures that $h(t) = 0$ at T-second intervals and the remaining terms in equation (19.2) reduce the tailing effect. A bandwidth comparison between the brickwall filter and the raised-cosine filter is shown in Table 19.1.

Filter	Filter bandwidth
Brickwall filter	$= f_c = 1/2T$
Raised-cosine filter	$= 2f_c = 1/T$

Table 19.1 Bandwidth of brickwall filter and raised-cosine filter.

It can be seen that, for the same transmission rate of $1/T$ pulses per second, the use of a raised-cosine filter doubles the bandwidth requirement.

Example 19.1

Sampling rate = 8000 samples per second.
Sampling interval $T = 1/8000$ seconds per sample.
Using a brickwall filter, required bandwidth = $1/2T$ = 4000 Hz.
Using a raised-cosine filter, required bandwidth = $1/T$ = 8000 Hz.

Theorem 19.1 If

$$H(f) = \begin{cases} \Pi\left(\frac{f}{2f_c}\right) + Y(f), & f < 2f_c \\ 0, & elsewhere \end{cases} \qquad (19.3)$$

where $Y(f)$ is a real function and has an even symmetry about $f = 0$, and $Y(f)$ has an odd symmetry about $f = f_c$, there will be no intersymbol interference at the sampling instants.

This theorem is illustrated in Figure 19.5 and the filter with a transfer function given by equation (19.3) is called a *Nyquist filter*.

Figure 19.5 Nyquist filter characteristic.

Proof. [2]

Taking the inverse Fourier transform of equation (19.3), we get

$$\begin{aligned} h(t) &= \int_{-2f_c}^{-f_c} Y(f)e^{j2\pi ft}df + \int_{-f_c}^{f_c} [1+Y(f)]e^{j2\pi ft}df + \int_{f_c}^{2f_c} Y(f)e^{j2\pi ft}df \\ &= \int_{-f_c}^{f_c} e^{j2\pi ft}df + \int_{-2f_c}^{2f_c} Y(f)e^{j2\pi ft}df \\ &= 2f_c\left(\frac{\sin 2\pi f_c t}{2\pi f_c t}\right) + \int_{-2f_c}^{0} Y(f)]e^{j2\pi ft}df + \int_{0}^{2f_c} Y(f)e^{j2\pi ft}df \end{aligned}$$

Letting $f_1 = f + f_c$ in the first integral and $f_1 = f - f_c$ in the second integral, we obtain

$$\begin{aligned} &= 2f_c\left(\frac{\sin 2\pi f_c t}{2\pi f_c t}\right) + e^{-j2\pi f_c t}\int_{-f_c}^{f_c} Y(f_1 - f_c)e^{j2\pi f_1 t}df_1 + \\ &\quad e^{-j2\pi f_c t}\int_{-f_c}^{f_c} Y(f_1 + f_c)e^{j2\pi f_1 t}df_1 \end{aligned}$$

$$= \; 2f_c\left(\frac{\sin 2\pi f_c t}{2\pi f_c t}\right) - e^{-j2\pi f_c t}\int_{-f_c}^{f_c} Y(f_1 + f_c)e^{j2\pi f_1 t}df_1 + e^{-j2\pi f_c t}\int_{-f_c}^{f_c} Y(f_1 + f_c)e^{j2\pi f_1 t}df_1$$

Since $Y(f = f_1)$ has an even symmetry about $f_1 = 0$ and an odd symmetry about $f_1 = f_c$, shifting $Y(f_1)$ to the left by f_c, we obtain $Y(f_1 - f_c)$, and $Y(f_1 - f_c)$ has an odd symmetry about $f_1 = 0$. Similarly, shifting $Y(f_1)$ to the right by f_c, we obtain $Y(f_1 + f_c)$, and $Y(f_1 + f_c)$ has an odd symmetry about $f_1 = 0$. For $-f_c \le f_1 \le f_c$, $Y(f_1 - f_c) = -Y(f_1 + f_c)$. We can write

$$h(t) = 2f_c\left(\frac{\sin 2\pi f_c t}{2\pi f_c t}\right) - j\,2\sin 2\pi f_c t\int_{-f_c}^{f_c} Y(f_1 + f_c)e^{j2\pi f_1 t}df_1$$

The sin $2\pi f_c t$ factor guarantees that $h(t) = 0$ at $t = \frac{n}{2f_c} = nT$ for $n \neq 0$ and $T = 1/2f_c$. Thus, if we sample at $t = \frac{n}{2f_c}$, there will be no ISI. □

An interesting transfer function satisfying Theorem 19.1 is the *sinusoidal roll-off spectrum* of Figure 19.6.

Figure 19.6 Sinusoidal roll-off spectrum.

If the desired pulse rate is $1/T$ pulses per second, the required bandwidth is

$$B = \frac{1}{2T}\left(1+\frac{f_x}{f_c}\right) \qquad (19.4)$$

$$= \frac{1}{2T}(1+r) \qquad (19.5)$$

where $r = f_x/f_c$ is called the *roll-off factor* and B is referred to as the *Nyquist bandwidth*. When $r = 0$, we have a brickwall filter. When $r = 1$, we have a raised-cosine filter.

The number of pulses per second that may be transmitted is given by

$$\frac{1}{T} = \frac{2B}{(1+r)} \tag{19.6}$$

Symbol Rate, Bit Rate, and Bandwidth Efficiency

So far we have spoken of the number of pulses per second that can be sent over a channel of bandwidth B Hz. We shall use the term *symbols* rather than pulses from now on. The number of symbols per second is called the *symbol rate*. It is the symbol rate that determines the transmission bandwidth. Supposing that a symbol can take on M possible levels and we send $2B$ symbols per second over a B Hz bandlimited channel, the transmission *bit rate* is $2B\log_2 M$ bits per second. The *bandwidth efficiency* is 2 $\log_2 M$ bits per second per Hz. If B is fixed, we can increase the bit rate by simply increasing the value of M.

Eye Patterns [3-5]

In digital communication, eye patterns (diagrams) are widely used as a qualitative/visual performance indicator of a system. An eye diagram is a plot/trace of consecutive sections of a signal superimposed on a normalised time scale. By superimposing sections of fixed time length, the effects of interference and jitter can be determined. Figure 19.7 shows the eye pattern for binary signals.

Figure 19.7 Interpretation of the eye pattern for binary signals.

Specifically, we make the following statements:

1. The eye opening is defined as the distance from the decision threshold to the closest trace at the sampling instant.

2. The height of the eye opening, at a specified sampling time, defines the noise margin of the system.

3. The vertical trace width at the best sampling instant shows the amount of amplitude distortion (ISI).

4. The horizontal trace width around the zero crossings shows the amount of timing jitter.

5. Non-symmetries in the eye diagram indicate the presence of non-linearities.

In the case of M-ary signals, the eye pattern contains (M - 1) eye openings stacked up vertically one on the other. Figure 19.8 shows the eye pattern for 3-level signals.

Figure 19.8 Eye pattern for 3-level signals.

References

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[2] L. W. Couch, II, Digital and Analog Communication Systems, 6/e, Prentice Hall, 2001.

[3] S. Haykin, Communication Systems, 4/e, J. Wiley & Sons, 2001.

[4] K. Feher *et al.*, Telecommunications Measurements, Analysis, and Instrumentation, Prentice-Hall, 1987.

[5] Bell Telephone Laboratories, Transmission Systems for Communications, 5/e, 1982.

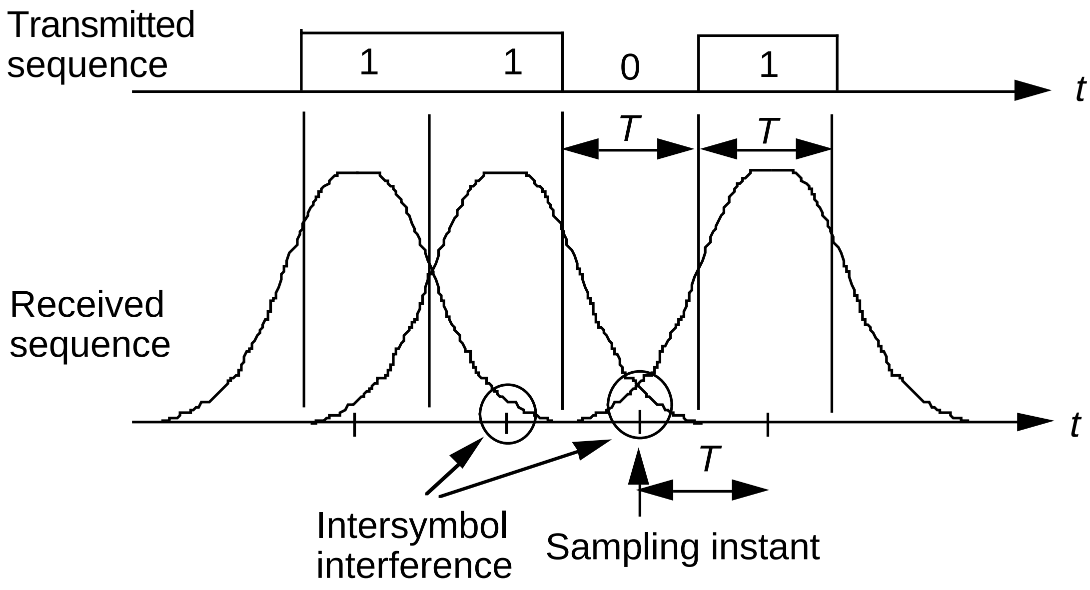

Figure 19.1 Intersymbol interference in digital baseband transmission.

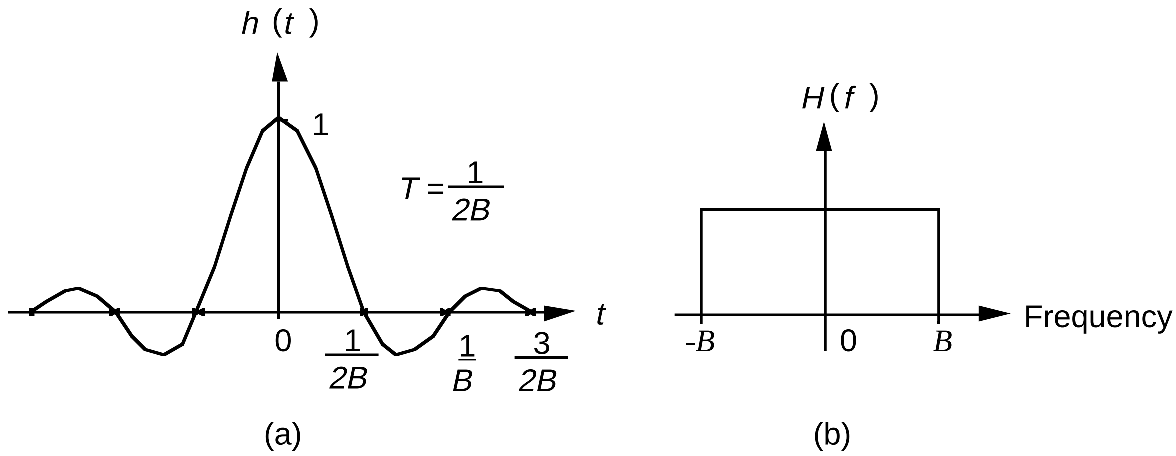

Figure 19.2 Pulse providing zero intersymbol interference.

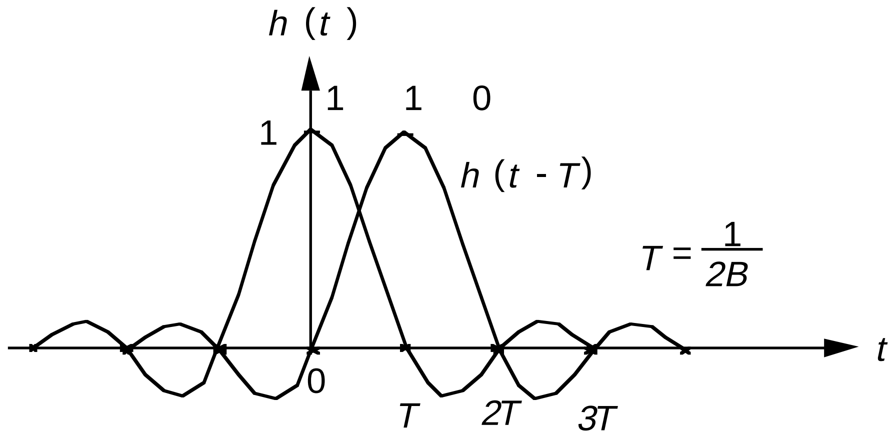

Figure 19.3 Transmission of pulses without intersymbol interference.

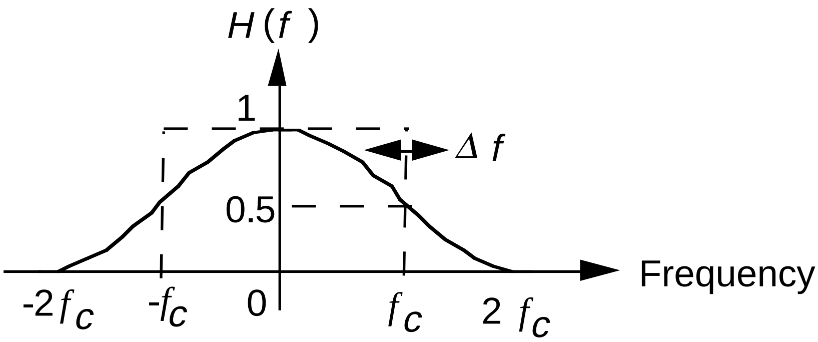

Figure 19.4 Raised-cosine filter spectrum.

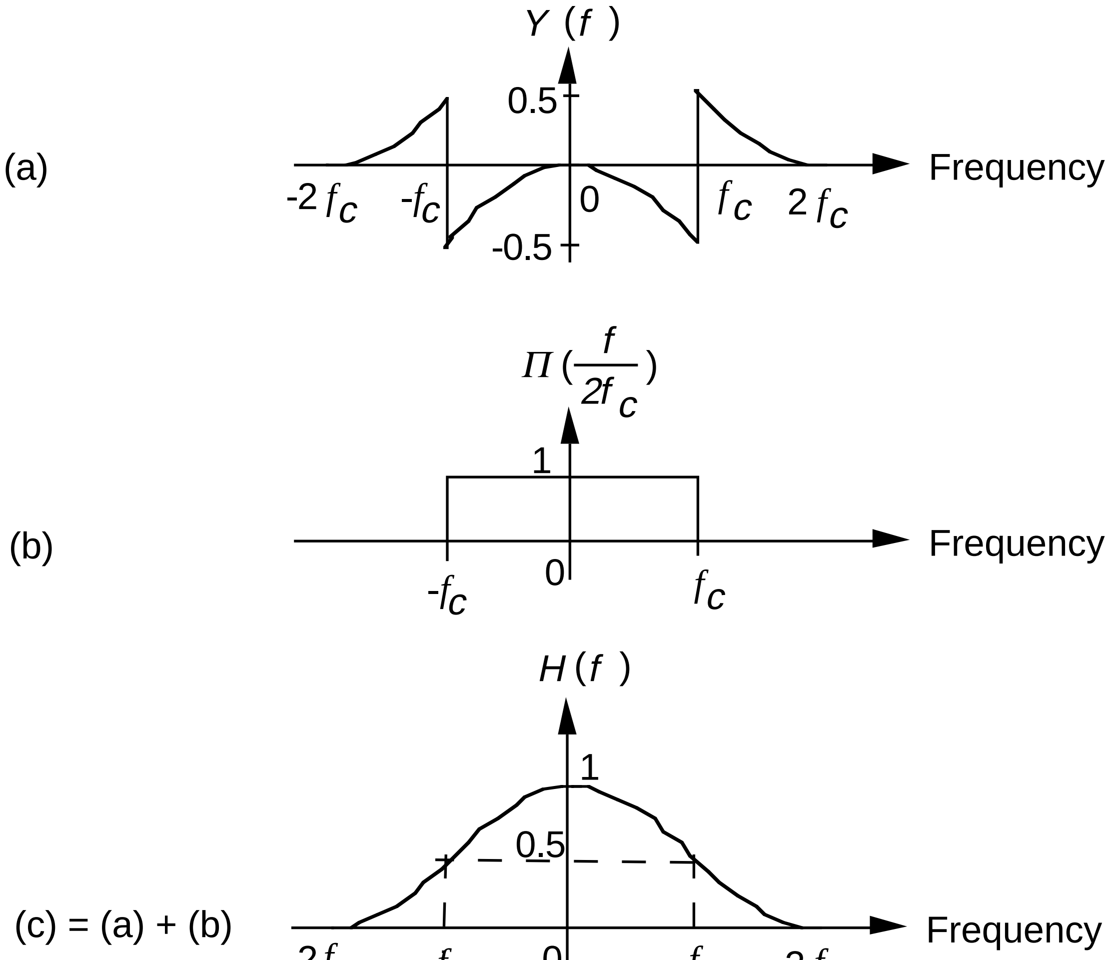

Figure 19.5 Nyquist filter characteristic.

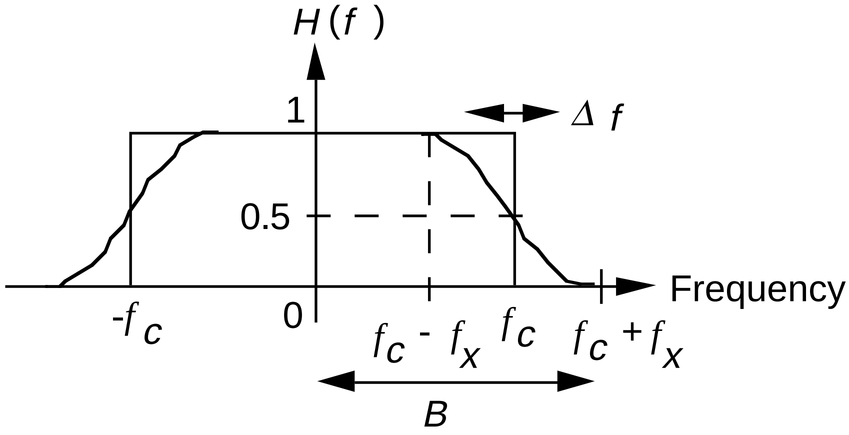

Figure 19.6 Sinusoidal roll-off filter spectrum.

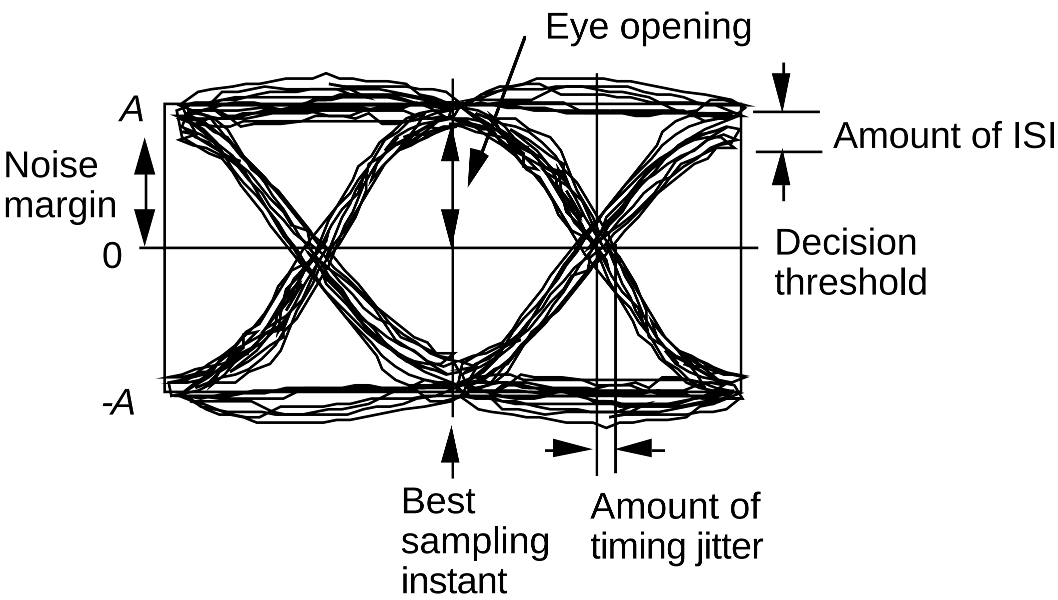

Figure 19.7 Interpretation of the eye pattern for binary signals.

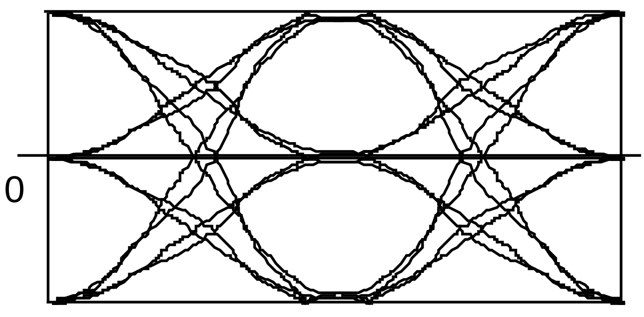

Figure 19.8 Eye pattern for 3-level signals.