1. Introduction to Signals and Operations

Model of a Communication System [1]

Figure 1.1 (a) Model of a communication system, and (b) signal processing functions.

Classification of Signals

1. Continuous-time and discrete-time signals [2]

By the term *continuous-time* signal we mean a real or complex function of time $s(t)$, where the independent variable t is continuous.

If t is a discrete variable, i.e., $s(t)$ is defined at discrete times, then the signal $s(t)$ is a *discrete-time* signal. A discrete-time signal is often identified as a sequence of numbers, denoted by $\{s(n)\}$, where n is an integer.

2. Analogue and digital signals [2]

If a continuous-time signal $s(t)$ can take on any values in a continuous time interval, then $s(t)$ is called an *analogue* signal.

If a discrete-time signal can take on only a finite number of distinct values, $\{s(n)\}$, then the signal is called a *digital* signal.

3. Deterministic and random signals [2]

Deterministic signals are those signals whose values are completely specified for any given time.

Random signals are those signals that take random values at any given times.

4. Periodic and nonperiodic signals [2]

A signal $s(t)$ is a *periodic* signal if $s(t) = s(t + nT_0)$, where T_0 is called the *period* and the integer $n > 0$.

If $s(t) \neq s(t + T_0)$ for all t and any T_0, then $s(t)$ is a *nonperiodic* or *aperiodic* signal.

5. Power and energy signals [2, 3]

A complex signal $s(t)$ is a *power* signal if *the average normalised power* P is finite, where

$$0 < P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} s(t)s^*(t)dt < \infty \tag{1.1}$$

and $s^*(t)$ is the complex conjugate of $s(t)$.

A complex signal $s(t)$ is an *energy* signal if *the normalised energy* E is finite, where

$$0 < E = \int_{-\infty}^{\infty} s(t)s^*(t)\, dt = \int_{-\infty}^{\infty} |\, s(t)\,|^2\, dt < \infty \tag{1.2}$$

In communication systems, the received waveform is usually categorised into the desired part, containing the information *signal*, and the undesired part, called *noise*.

Some Useful Functions

1. Unit impulse function [2]

The *unit impulse function*, also known as the *dirac delta function*, $\delta(t)$, is defined by

$$\int_{-\infty}^{\infty} s(t)\, \delta(t)\, dt = s(0) \tag{1.3}$$

An alternative definition [4] is

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \tag{1.4a}$$

and

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases} \tag{1.4b}$$

Figure 1.2 Unit impulse function.

2. Unit step function [2]

The *unit step function* $u(t)$ is

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases} \tag{1.5}$$

Figure 1.3 Unit step function.

and the unit step function is related to the unit impulse function by

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau \tag{1.6}$$

and

$$\frac{du(t)}{dt} = \delta(t) \tag{1.7}$$

3. Sampling function [4]

A *sampling function* is denoted by

$$\mathrm{Sa}(x) = \frac{\sin x}{x} \tag{1.8}$$

Figure 1.4 Sampling function.

4. Sinc function [4]

A *sinc function* is denoted by

$$\mathrm{sinc}\, x = \frac{\sin \pi x}{\pi x} \tag{1.9}$$

Hence,

$$\mathrm{Sa}(x) = \mathrm{sinc}\left(\frac{x}{\pi}\right) \tag{1.10}$$

5. Rectangular function [4]

A single *rectangular pulse* is denoted by

$$\Pi\left(\frac{t}{T}\right) = \begin{cases} 1, & |t| \leq \frac{T}{2} \\ 0, & |t| > \frac{T}{2} \end{cases} \tag{1.11}$$

6. Triangular function [4]

A *triangular function* is denoted by

$$\Lambda\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ 0, & |t| > T \end{cases} \tag{1.12}$$

Some Useful Operations [4]

1. Time average

The *time average operator* is given by

$$\langle[.]\rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} [.]dt \tag{1.13}$$

If a waveform is periodic, the time average operator can be reduced to

$$\langle[.]\rangle = \frac{1}{T_0} \int_{a-T_0/2}^{a+T_0/2} [.]dt \tag{1.14}$$

where T_0 is the period of the waveform and a is an arbitrary real constant, which may be taken to be zero. Equation (1.14) readily follows from (1.13) because, referring to (1.13), integrals over successive time intervals T_0 seconds wide have identical area if the waveform is periodic. As these integrals are summed, the total area and T are proportionally larger, resulting in a value for the time average that is the same as just integrating over one period and dividing by T_0.

In summary, (1.13) may be used to evaluate the time average of any type of waveform.

Equation (1.14) is valid only for periodic waveforms.

2. Direct-current value

The *direct-current (dc)* value of a waveform is given by

$$<s(t)> = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} s(t)\,dt \tag{1.15}$$

We can see that this is the time average of $s(t)$. Over a finite interval of interest, the dc value is

$$<s(t)> = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} s(t)\,dt \tag{1.16}$$

3. Power and energy

The *instantaneous power* (incremental work divided by incremental time) is given by

$$p(t) = v(t)\, i(t) \tag{1.17}$$

where $v(t)$ denotes voltage and $i(t)$ denotes current.

The *average power* is given by

$$P = <p(t)> = <v(t)\, i(t)> \tag{1.18}$$

The *root mean square (rms)* value of $s(t)$ is given by

$$S_{rms} = \sqrt{<s^2(t)>} \tag{1.19}$$

If a load is resistive, the average power is given by

$$P = \frac{<v^2(t)>}{R} = <i^2(t)>R = \frac{V_{rms}^2}{R} = I_{rms}^2 R = V_{rms} I_{rms} \tag{1.20}$$

where R is the value of the resistive load. When $R = 1\ \Omega$, P becomes the *normalised* *power*.

The *average normalised power* of a real-valued signal $s(t)$ is given by

$$P = \langle s^2(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} s^2(t)dt \tag{1.21}$$

The *total normalised energy* of a real-valued signal $s(t)$ is given by

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} s^2(t)dt \tag{1.22}$$

4. Decibel

The *decibel gain* of a circuit is given by

$$\text{dB} = 10 \log_{10}\left(\frac{P_{out}}{P_{in}}\right) \tag{1.23}$$

If resistive loads are involved, (1.23) can be reduced to

$$\text{dB} = 20 \log_{10}\left(\frac{V_{rms\ out}}{V_{rms\ in}}\right) + 10 \log_{10}\left(\frac{R_{in}}{R_{load}}\right) \tag{1.24}$$

or

$$\text{dB} = 20 \log_{10}\left(\frac{I_{rms\ out}}{I_{rms\ in}}\right) + 10 \log_{10}\left(\frac{R_{load}}{R_{in}}\right) \tag{1.25}$$

If normalised powers are used,

$$\text{dB} = 20 \log_{10}\left(\frac{V_{rms\ out}}{V_{rms\ in}}\right) = 20 \log_{10}\left(\frac{I_{rms\ out}}{I_{rms\ in}}\right) \tag{1.26}$$

The *decibel power level* with respect to 1 mW is given by

$$\text{dBm} = 10\log_{10}\left(\frac{actual\ \ power\ \ level\ \ in\,watts}{10^{-3}}\right) \tag{1.27}$$

The *decibel power level* with respect to 1 W is given by

$$\text{dBW} = 10\log_{10}(\text{actual power level in watts})$$

The *decibel power level* with respect to a 1 mV rms level is given by

$$\text{dBmV} = 20\log_{10}\left(\frac{V_{rms}}{10^{-3}}\right) \tag{1.28}$$

5. Complex number and phasor

A complex number c is said to be a *phasor* if it is used to represent a *sinusoidal* waveform. That is,

$$s(t) = |c|\cos[\omega_0 t + \angle c] = \text{Re}\,\{c\, e^{j\omega_0 t}\} \tag{1.29}$$

where the phasor

$$c = |c|\, e^{j\phi} \tag{1.30}$$

and $\phi = \angle c$.

Other Useful Operations

1. Cross-correlation [5]

The *cross-correlation* of two real-valued power waveforms $s_1(t)$ and $s_2(t)$ is defined by

$$R_{12}(\tau) = \langle s_1(t)\, s_2(t+\tau)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} s_1(t)\, s_2(t+\tau)\, dt \tag{1.31}$$

If $s_1(t)$ and $s_2(t)$ are periodic with the same period T_0, then

$$R_{12}(\tau) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} s_1(t)\, s_2(t+\tau)\, dt \qquad (1.32)$$

The cross-correlation of two real-valued energy waveforms $s_1(t)$ and $s_2(t)$ is defined by

$$R_{12}(\tau) = \int_{-\infty}^{\infty} s_1(t)\, s_2(t+\tau)\, dt \qquad (1.33)$$

Correlation is a useful operation to measure the similarity between two waveforms. To compute the correlation between waveforms, it is necessary to specify which waveform is being shifted. In general, $R_{12}(\tau)$ is not equal to $R_{21}(\tau)$, where $R_{21}(\tau) = \langle s_2(t)\, s_1(t+\tau)\rangle$.

The cross-correlation of two complex waveforms is $R_{12}(\tau) = \langle s_1^*(t)\, s_2(t+\tau)\rangle$.

2. Auto-correlation [2, 4]

The *auto-correlation* of a real-valued power waveform $s_1(t)$ is defined by

$$R_{11}(\tau) = \langle s_1(t)\, s_1(t+\tau)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} s_1(t)\, s_1(t+\tau)\, dt \qquad (1.34)$$

If $s_1(t)$ is periodic with fundamental period T_0, then

$$R_{11}(\tau) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} s_1(t)\, s_1(t+\tau)\, dt \qquad (1.35)$$

The auto-correlation of a real-valued energy waveform $s_1(t)$ is defined by

$$R_{11}(\tau) = \int_{-\infty}^{\infty} s_1(t)\, s_1(t+\tau)\, dt \qquad (1.36)$$

The auto-correlation of a complex power waveform is $R_{11}(\tau) = \langle s_1{}^*(t)s_1(t+\tau)\rangle$.

3. Convolution [4]

The *convolution* of a waveform $s_1(t)$ with a waveform $s_2(t)$ is given by

$$s_3(t) = s_1(t) * s_2(t) = \int_{-\infty}^{\infty} s_1(\lambda)\, s_2(t-\lambda)\, d\lambda \tag{1.37a}$$

$$s_3(t) = s_1(t) * s_2(t) = \int_{-\infty}^{\infty} s_1(\lambda)\, s_2[-(\lambda-t)]\, d\lambda \tag{1.37b}$$

where * denotes the convolution operation. (1.37b) is obtained by

1. Time reversal of $s_2(t)$ to obtain $s_2(-\lambda)$.
2. Time shifting of $s_2(-\lambda)$ to obtain $s_2[-(\lambda-t)]$.
3. Multiplying $s_1(\lambda)$ and $s_2[-(\lambda-t)]$ to form the integrand $s_1(\lambda)\, s_2[-(\lambda-t)]$.

Example 1.1 Convolution of a rectangular waveform $s_1(t) = \begin{cases} 1, & 0 < t < T \\ 0, & elsewhere \end{cases}$ with an exponential waveform $s_2(t) = e^{-t/T}u(t)$.

Figure 1.5 Convolution between a rectangular waveform and an exponential waveform.

References

[1] B. Sklar, Digital Communications, Prentice Hall, 1998.

[2] H. P. Hsu, Analog and Digital Communications, McGraw-Hill, 1993.

[3] J. D. Gibson, Modern Digital and Analog Communications, 2/e, Macmillan Publishing Company, 1993.

[4] L. W. Couch II, Digital and Analog Communication Systems, 5/e, Prentice Hall, 1997.

[5] H. Taub, and D. L. Schilling, Principles of Communication Systems, 2/e, McGraw-Hill, 1986.

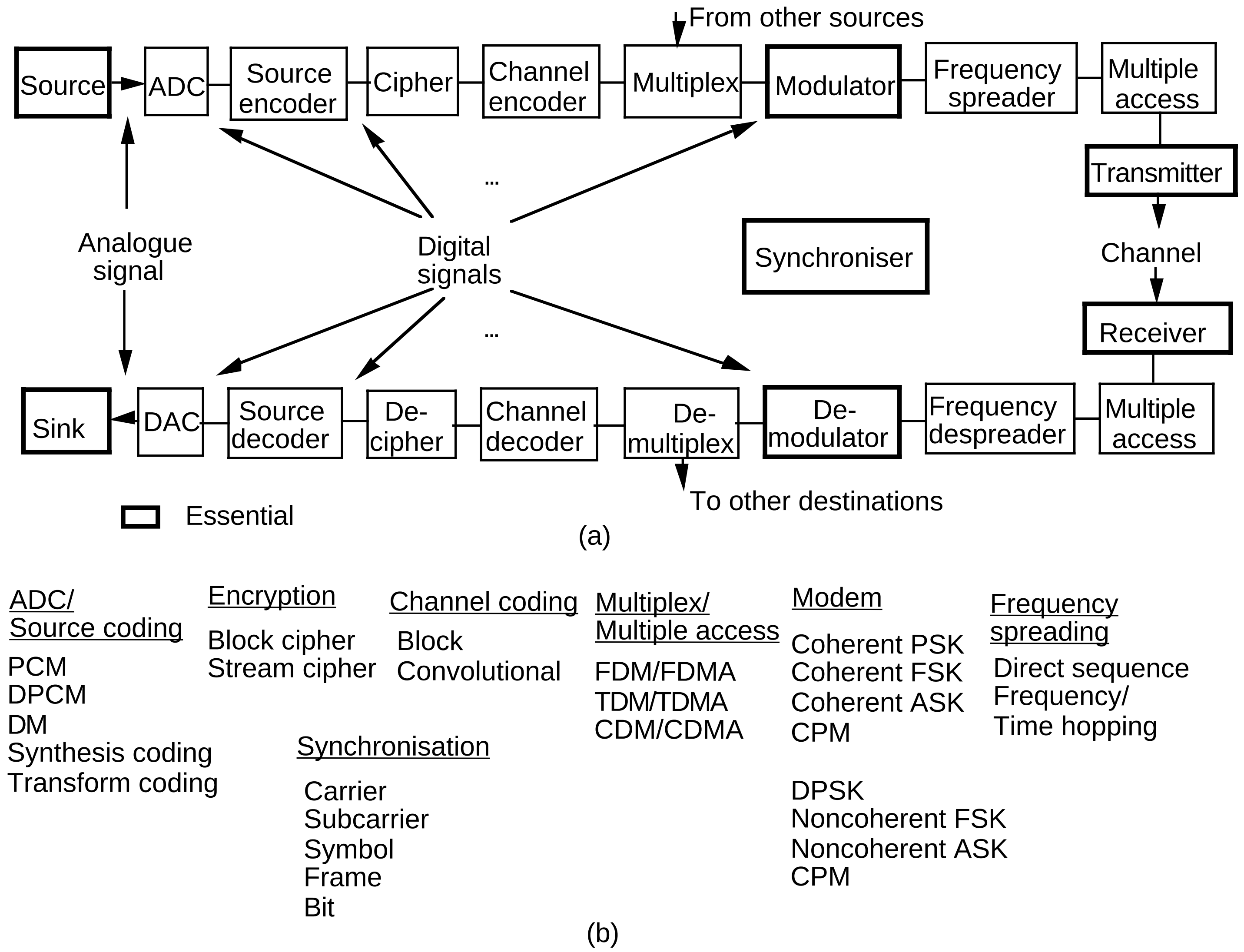


Figure 1.1 (a) Model of a communication system, and (b) signal processing functions.

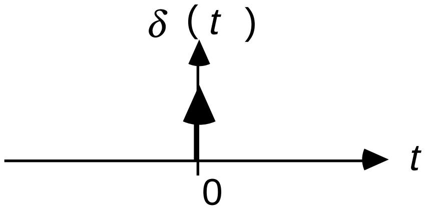


Figure 1.2 Unit impulse function.

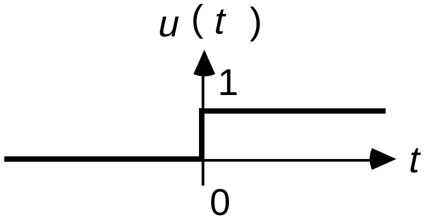


Figure 1.3 Unit step function.

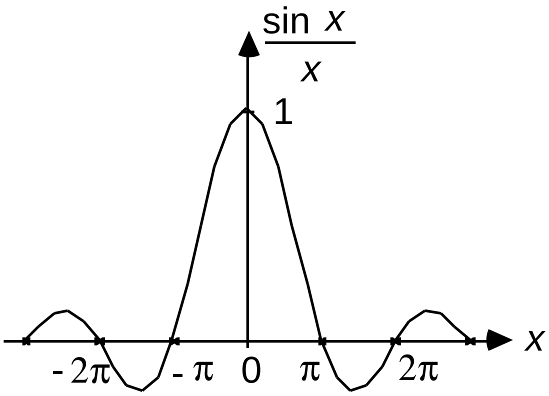
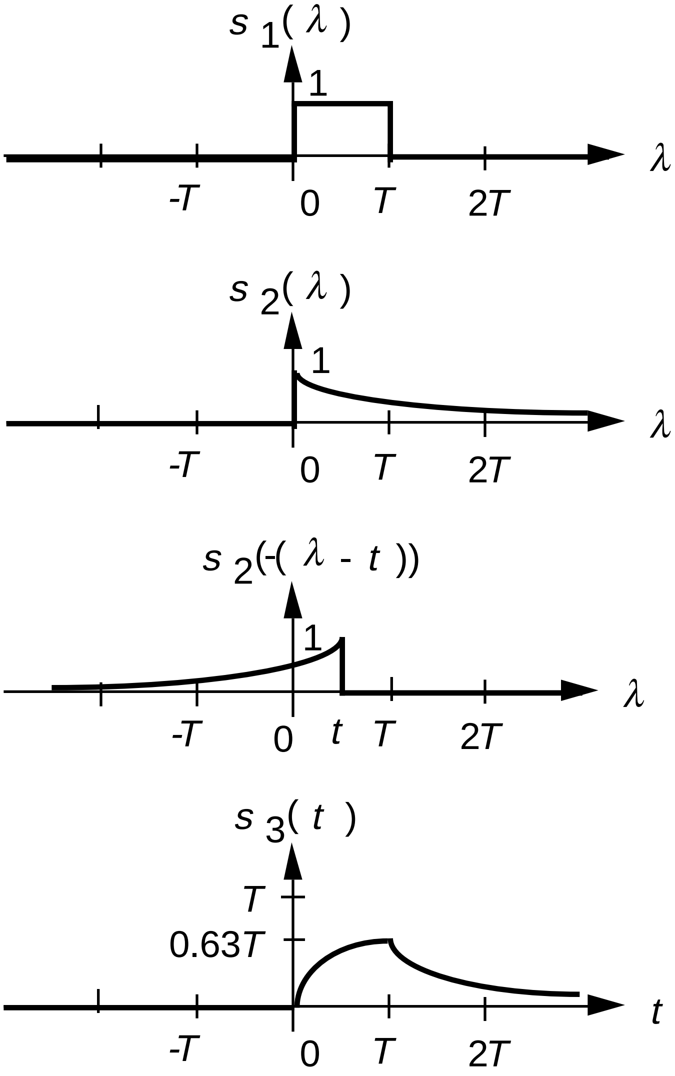


Figure 1.4 Sampling function.



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