8. Single-Sideband and Vestigial-Sideband Modulations

We have seen that both normal AM and DSB signals require a transmission bandwidth equal to twice the bandwidth of the message signal $m(t)$. Since either the upper sideband (USB) or the lower sideband (LSB) contains the complete information of the message signal, we can conserve bandwidth by transmitting only one sideband. The modulation is called *single-sideband (SSB) modulation*. It is widely used by the military and by radio amateurs in *high-frequency (HF)* communication systems.

Generation of SSB Signals

There are two common methods to generate a single-sideband signal.

Filter Method.

In this method, a balanced modulator is used to generate a DSB signal, and the desired sideband signal is then selected by a bandpass filter for transmission. Figures 8.1 and 8.2 show the generation of a SSB signal using the filter method, and the spectra associated with it.

Figure 8.1 Generation of SSB signal using filter method.

Figure 8.2 Spectra associated with SSB signal using filter method.

The technique is suitable for message signals with very little frequency content down to dc and hence does not require sharp filter cut-off characteristics.

Phasing Method.

A single-sideband signal is given by

$$s_c(t) = m(t) \cos 2\pi f_c t \mp \hat{m}(t) \sin 2\pi f_c t \qquad (8.1)$$

where the - sign is associated with the USB and the + sign is associated with the LSB. $\hat{m}(t)$ is obtained by shifting the phase of all frequency components of $m(t)$ by -90°. $\hat{m}(t)$ is called the *Hilbert transform* of $m(t)$ and is defined as

$$\hat{m}(t) = H[m(t)] = m(t) * h(t) = m(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau} d\tau \qquad (8.2)$$

where $h(t) = \frac{1}{\pi t}$. The inverse Hilbert transform of $\hat{m}(t)$ is

$$m(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\hat{m}(\tau)}{t-\tau}d\tau \qquad (8.3)$$

Proof [1].

Let $M(f)$ be the Fourier transform of $m(t)$. Defining

$$\begin{cases} M_+(f) = M(f) & f \geq 0 \\ M_-(f) = M(f) & f < 0 \end{cases} \qquad (8.4)$$

we can write

$$M(f) = M_+(f) + M_-(f) \qquad (8.5)$$

Consider a linear time-invariant circuit which shifts the phase of all positive-frequency components of an input signal $m(t)$ by -90^o and all negative-frequency components of $m(t)$ by +90^o. This is shown in Figure 8.3.

Figure 8.3 Block diagram of a 90^o phase-shift network.

Let $\hat{M}(f)$ be the Fourier transform of $\hat{m}(t)$. We have

$$\begin{cases} \hat{M}(f) = -jM(f) & f \geq 0 \\ \hat{M}(f) = jM(f) & f < 0 \end{cases} \qquad (8.6)$$

Substituting equation (8.4) into (8.6), we get

$$\hat{M}(f) = -j\,M_+(f) + j\,M_-(f) \qquad (8.7)$$

Taking the Fourier transform of equation (8.1), we have

$$S_c(f) = \frac{1}{2}[M(f-f_c) + M(f+f_c)] \mp \frac{1}{2j}[\hat{M}(f-f_c) - \hat{M}(f+f_c)] \qquad (8.8)$$

Substituting equations (8.5) and (8.7) into equation (8.8), we obtain

$$S_c(f) = \frac{1}{2}[M_+(f-f_c) + M_-(f-f_c) + M_+(f+f_c) + M_-(f+f_c)] \mp$$

$$\frac{1}{2j}[-j\,M_+(f-f_c) + j\,M_-(f-f_c) + j\,M_+(f+f_c) - j\,M_-(f+f_c)]$$

$$S_c(f) = \frac{1}{2}[M_+(f-f_c) + M_-(f-f_c) + M_+(f+f_c) + M_-(f+f_c)] \mp \frac{1}{2}[-M_+(f-f_c) + M_-(f-f_c) + M_+(f+f_c) - M_-(f+f_c)]$$

$$S_c(f) = \begin{cases} M_+(f-f_c) + M_-(f+f_c) & \text{for } LSB \\ M_+(f+f_c) + M_-(f-f_c) & \text{for } USB \end{cases} \tag{8.9}$$

□

Figures 8.4 and 8.5 show the generation of a SSB signal using the phasing method, and the spectra associated with it.

Figure 8.4 Generation of SSB signal using phasing method.

Figure 8.5 Spectra of SSB signal using phasing method.

Advantages:

1. Does not employ bandpass filter.
2. Suitable for message signals with frequency content down to dc.

Disadvantage:

Practical realisation of a wideband 90^o phase shift circuit is difficult.

Demodulation of SSB Signals

Demodulation of SSB signals can be accomplished by using a synchronous detector as used in the demodulation of normal AM and DSB signals. This is shown in Figure 8.6.

Figure 8.6 Synchronous detector.

At the receiving end, the bandpass signal is multiplied by a locally generated carrier signal $\cos 2\pi f_c t$, which is in synchronism with the transmitted carrier signal. The output of the multiplier is

$$\begin{aligned} x(t) &= [m(t)\cos 2\pi f_c t \mp \hat{m}(t)\sin 2\pi f_c t]\cos 2\pi f_c t \\ &= m(t)\cos^2 2\pi f_c t \mp [\hat{m}(t)\sin 2\pi f_c t]\cos 2\pi f_c t \\ &= 0.5\,m(t) + 0.5\,m(t)\cos 4\pi f_c t \mp 0.5\,\hat{m}(t)\sin 4\pi f_c t \end{aligned} \tag{8.10}$$

If we suppress the last two terms using a low-pass filter, we get

$$y(t) = 0.5\, m(t) \tag{8.11}$$

That is, we can recover the component $m(t)$. If the carrier signal has phase or frequency errors, the recovered message is distorted.

If we want to use an envelope detector, it can be shown that we must insert a pilot carrier signal $A \cos 2\pi f_C t$ to the SSB signal, where $A >> m(t)$ and $A >> \hat{m}(t)$ [1]. The pilot signal carries most of the transmission power and the transmission becomes very inefficient.

Vestigial Sideband (VSB) Modulation [1, 2]

Vestigial sideband modulation is a compromise between DSB and SSB modulations. It relaxes the sharp cutoff requirement of a SSB signal by retaining a trace of the other sideband in the transmitted signal. Typically, the bandwidth of a VSB modulated signal is about 1.25 times that of the corresponding SSB modulated signal. It is commonly used for transmission of video signals in commercial television broadcasting. Figures 8.7 and 8.8 show the generation of a VSB signal, and the spectra associated with it.

Figure 8.7 Generation of VSB signal.

Figure 8.8 Spectra associated with VSB signal.

By inspection of Figure 8.8, the Fourier transform of a VSB modulated signal $s_C(t)$ is

$$S_C(f) = \frac{1}{2}[M(f-f_C) + M(f+f_C)]H(f) \tag{8.12}$$

where $H(f)$ is the transfer function of the bandpass filter.

Demodulation of VSB Signals

Demodulation of VSB signals can be accomplished by using a synchronous detector. Let $s_C(t)$ be the input signal to the synchronous detector. At the receiving end, the bandpass signal is multiplied by a locally generated carrier signal $\cos 2\pi f_C t$, which is in synchronism with the transmitted carrier signal. The output of the multiplier is

$$x(t) = s_C(t) \cos 2\pi f_C t \tag{8.13}$$

and the Fourier transform of $x(t)$ is

$$X(f) = \frac{1}{2}[S_c(f-f_c) + S_c(f+f_c)] \tag{8.14}$$

Substituting equation (8.12) into (8.14), we get

$$X(f) = \frac{1}{2}\{[\frac{1}{2}[M(f-2f_c) + M(f)]H(f-f_c)] + [\frac{1}{2}[M(f) + M(f+2f_c)]H(f+f_c)]\} \tag{8.15}$$

We can suppress the frequency components at $\pm 2f_c$ by a low-pass filter and we get

$$Y(f) = \frac{1}{4}M(f)[H(f-f_c) + H(f+f_c)] \tag{8.16}$$

For distortionless detection, we must have

$$H(f-f_c) + H(f+f_c) = K, \qquad |f| \leq B \tag{8.17}$$

where K is a constant and B is the bandwidth of the message signal. VSB modulated signals can also be detected by an envelope detector. As in the demodulation of a SSB signal, we need to send a pilot carrier signal, resulting an inefficient use of available transmitted power.

References

[1] J. D. Gibson, Modern Digital and Analog Communications, 2/e, Macmillan Publishing Company, 1993.

[2] H. P. Hsu, Analog and Digital Communications, McGraw-Hill, 1993.

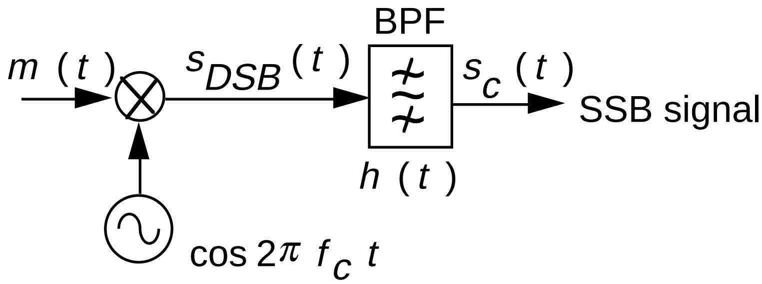

Figure 8.1 Generation of SSB signal using filter method.

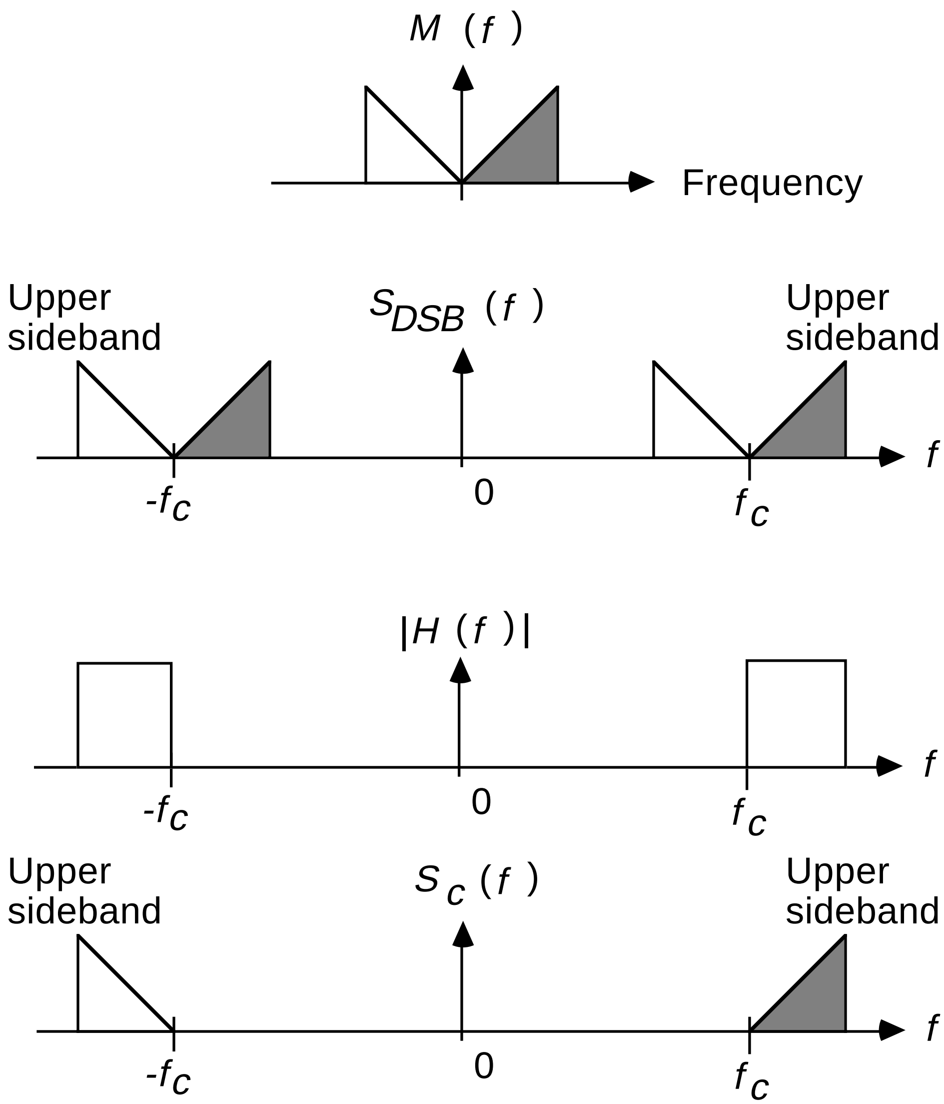

Figure 8.2 Spectra associated with SSB signal using filter method.

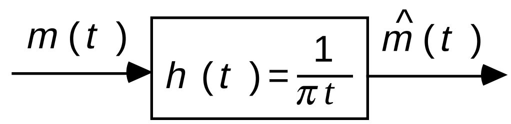

Figure 8.3 Block diagram of a 90^o phase-shift network.

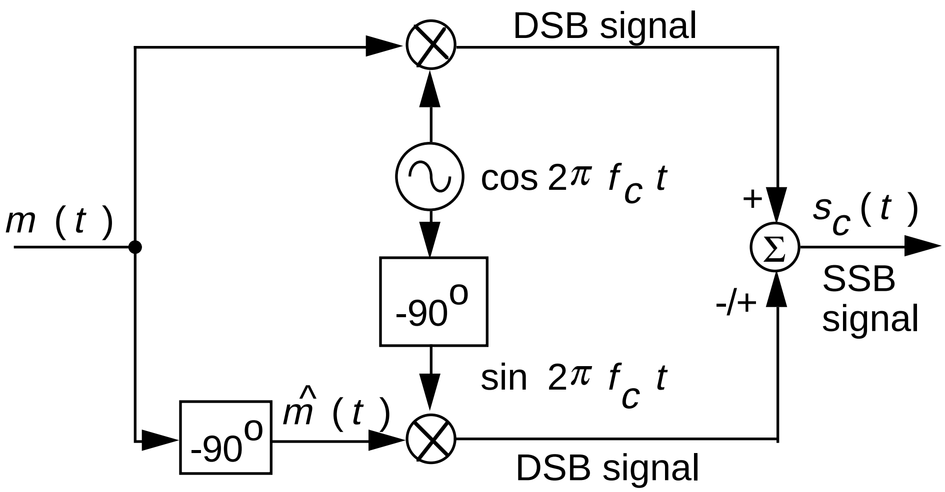

Figure 8.4 Generating SSB signal using phasing method.

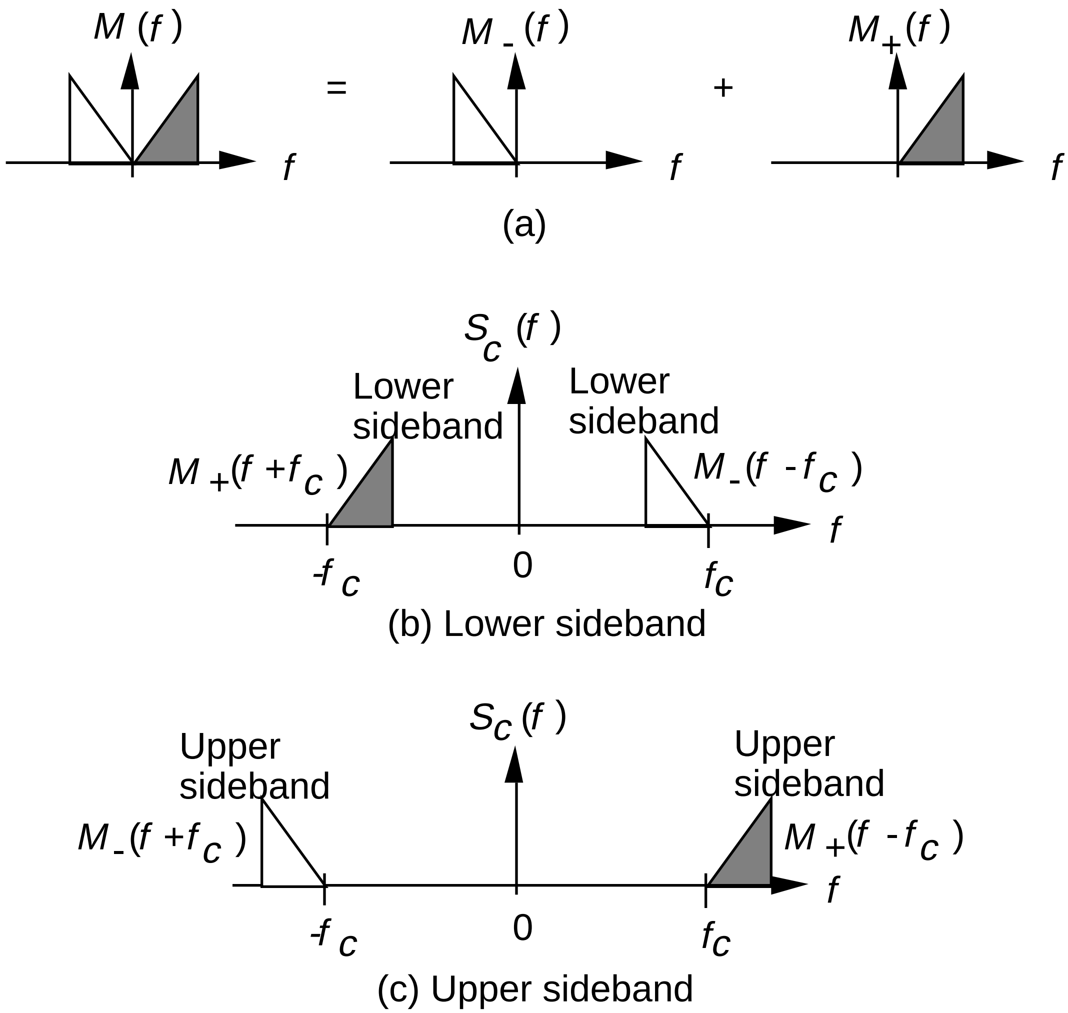

Figure 8.5 SSB spectra expressed in terms of $M_+(f)$ and $M_-(f)$.

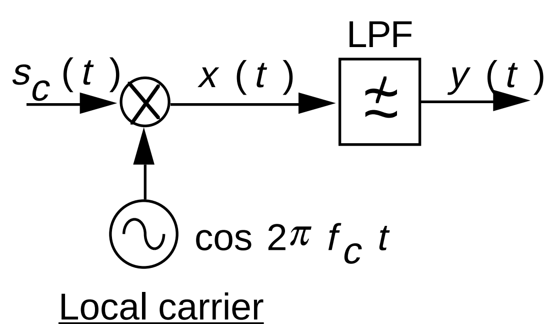

Figure 8.6 Synchronous detector.

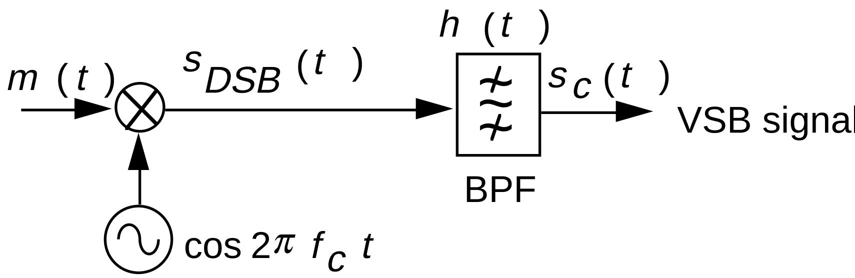

Figure 8.7 VSB modulator.

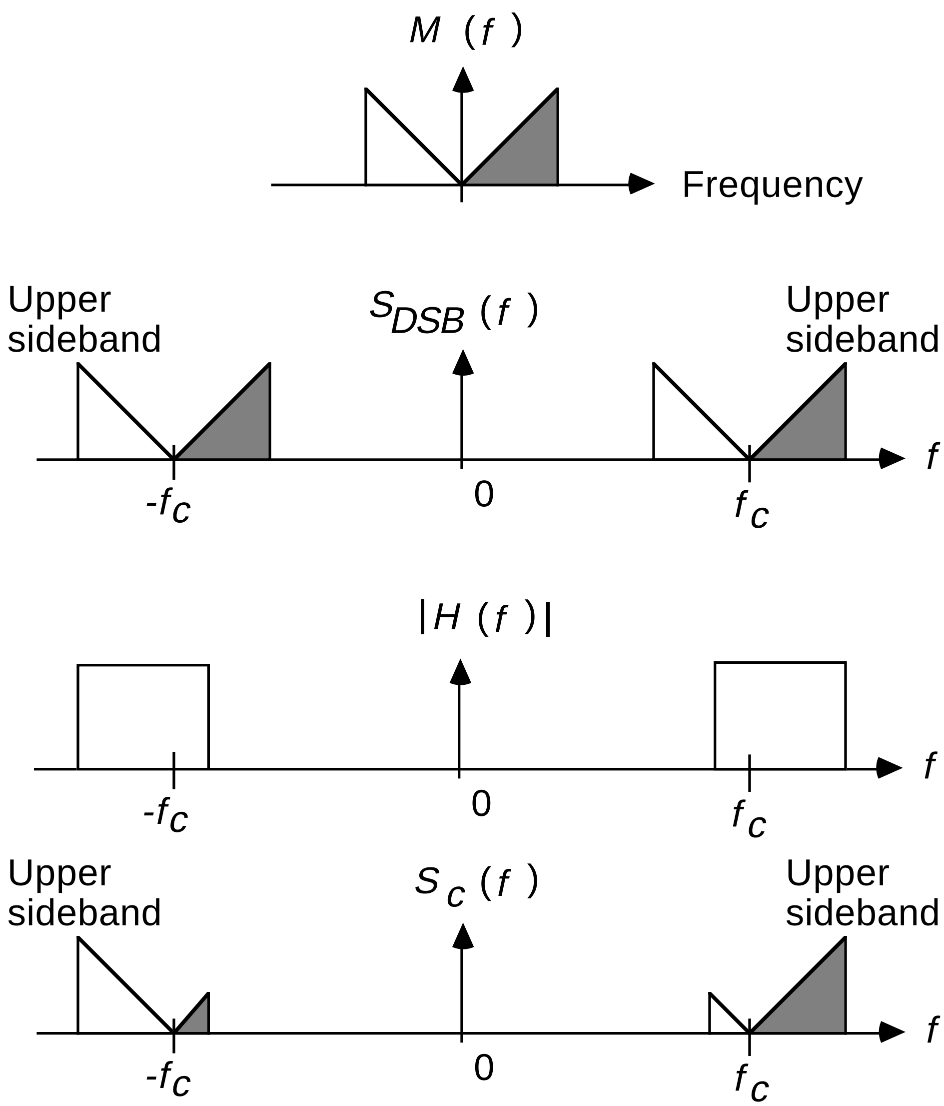

Figure 8.8 Spectra associated with VSB signal.