10. Wideband Frequency Modulation (WFM)

Review of Angle Modulation

Angle modulation encompasses phase modulation (PM) and frequency modulation (FM). We have seen that an angle-modulated signal can be represented by [1]

$$s_c(t) = A\cos\left[2\pi f_c t + \phi(t)\right] \tag{10.1}$$

where A is a constant. $\phi(t)$ is a function of the modulating signal and is given by

$$\phi(t) = \begin{cases} k_p m(t) & \text{for } PM \\ k_f \int_{-\infty}^{t} m(\lambda)\,d\lambda & \text{for } FM \end{cases} \tag{10.2}$$

Because of the difficulty of analysing general angle-modulated signals, we shall only consider angle-modulated signals with a sinusoidal modulating signal. Let the modulating signal be

$$m(t) = \begin{cases} a_m \sin 2\pi f_m t & \text{for } PM \\ a_m \cos 2\pi f_m t & \text{for } FM \end{cases} \tag{10.3}$$

Substituting (10.3) into (10.2), we have

$$\phi(t) = \beta \sin 2\pi f_m t \tag{10.4}$$

where

$$\beta = \begin{cases} k_p a_m & \text{for } PM \\ \dfrac{k_f a_m}{2\pi f_m} & \text{for } FM \end{cases} \tag{10.5}$$

For FM with a sinusoidal modulating signal, the frequency modulation index is

$$\beta = \frac{\Delta f}{B} \tag{10.6}$$

where $B = f_m$ is the bandwidth of the modulating signal and Δf is the peak frequency deviation. The peak frequency deviation is given by

$$\Delta f = \frac{1}{2\pi} k_f \max |m(t)| \qquad (10.7)$$

Wideband Frequency Modulation

Consider the angle-modulated signal $s_c(t) = A\cos\,(2\pi f_c t + \beta \sin\, 2\pi f_m t)$ with sinusoidal modulating signal $m(t) = a_m \cos 2\pi f_m t$. It can be shown that $s_c(t)$ can also be written as

$$s_c(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + nf_m)t \qquad (10.8)$$

where

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - n x)}\, dx \qquad (10.9)$$

The integral is known as the *Bessel function of the first kind of the n-th order* and cannot be evaluated in closed form. Figure 10.1 shows some Bessel functions for $n = 0$, 1, 2, 3, and 8. Clearly, the value of $J_n(\beta)$ becomes small for large values of n.

Figure 10.1 Bessel functions.

Proof [1].

$$s_c(t) = A\cos\,(2\pi f_c t + \beta \sin\, 2\pi f_m t)$$
$$s_c(t) = A\ \mathrm{Re}(e^{j2\pi f_c t}\, e^{j\beta \sin\, 2\pi f_m t}) \qquad (10.10)$$

Clearly, $e^{j\beta \sin\, 2\pi f_m t}$ is a periodic function with period $T_0 = 1/ f_m$, and has a Fourier series representation

$$e^{j\beta \sin\, 2\pi f_m t} = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} c_n\, e^{j2\pi\, n f_m t} \qquad (10.11)$$

The Fourier coefficient is

$$c_n = \int_{-T_0/2}^{T_0/2} e^{j(\beta \sin\, 2\pi f_m t\, -\, 2\pi\, n f_m t)}\, dt$$

Let $x = 2\pi f_m t$. Therefore,

$$c_n = \frac{1}{2\pi f_m} \int_{-\pi}^{\pi} e^{j(\beta \sin x - n x)}\, dx$$
$$= T_0 J_n(\beta) \qquad (10.12)$$

Substituting c_n into equation (10.11), we have

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi n f_m t} \qquad (10.13)$$

Substituting equation (10.13) into equation (10.10), we have

$$s_c(t) = A \text{ Re } [e^{j2\pi f_c t} \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi f_m t}]$$
$$= A \text{ Re } [\sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi (f_c + n f_m) t}]$$

Taking the real part yields $s_c(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + n f_m)t$. □

Also, it can be shown from the integral definition of $J_n(\beta)$ that

$$J_n(\beta) = \begin{cases} J_{-n}(\beta), & n \text{ even} \\ -J_{-n}(\beta), & n \text{ odd} \end{cases} \qquad (10.14)$$

Therefore, we can write

$$\begin{aligned} s_c(t) = \; & A\{J_0(\beta)\cos 2\pi f_c t \, - \\ & J_1(\beta)[\cos 2\pi(f_c - f_m)t - \cos 2\pi(f_c + f_m)t\,] + \\ & J_2(\beta)[\cos 2\pi(f_c - 2f_m)t + \cos 2\pi(f_c + 2f_m)t\,] - \\ & J_3(\beta)[\cos 2\pi(f_c - 3f_m)t - \cos 2\pi(f_c + 3f_m)t\,] + \ldots\} \end{aligned} \qquad (10.15)$$

Figure 10.2 shows the amplitude spectra of FM signals with a sinusoidal modulating signal and fixed f_m.

Figure 10.2 Amplitude spectra of FM signals with sinusoidal modulating signal and fixed f_m.

Observations:

1. The spectrum consists of a carrier component at f_c plus sideband components at $f_c \pm nf_m$ (n = 1, 2, ...).

2. The number of sideband terms depends on the modulation index β.

3. The magnitude of the carrier signal decreases rapidly as β increases.

4. The amplitudes of the spectral lines depend on the value of $J_n(\beta)$ (see equation (10.15)).

5. The bandwidth of the modulated signal with a sinusoidal modulating signal increases as β increases, and the bandwidth of the modulated signal is larger than $2\Delta f$.

Figure 10.3 shows the amplitude spectra of FM signals with a sinusoidal modulating signal and a fixed peak frequency deviation Δf [2]. Clearly, we get more and more spectral lines crowding into a fixed frequency interval as f_m decreases.

Figure 10.3 Amplitude spectra of FM signals with sinusoidal modulating signal and fixed peak frequency deviation Δf.

Bandwidth of Angle-Modulation Signals

From equation (10.15), we observe that the spectrum consists of a carrier component at f_c plus an infinite number of sideband components at $f_c \pm nf_m$ (n = 1, 2, ...). In fact, 98% of the normalised total signal power is contained in the bandwidth

$$B_T = 2(\beta + 1)\, B \qquad (10.16)$$

where β is either the phase modulation index or the frequency modulation index and B is the bandwidth of the modulating signal. The bandwidth of the angle-modulated signal with sinusoidal modulating signal depends on β and B. This is called *Carson's rule*. It gives a rule-of-thumb expression and an easy way to evaluate the transmission bandwidth of angle-modulated signals. When $\beta \ll 1$, the signal is a narrowband angle-modulated signal and its bandwidth is approximately equal to $2B$.

Generation of Wideband FM

Indirect Method.

In this method, a narrowband frequency-modulated signal is first generated using an integrator and a phase modulator. A frequency multiplier is then used to increase the peak frequency deviation from Δf to $n\Delta f$. Use of frequency multiplication normally increases the carrier frequency from f_c to $n\,f_c$. A mixer or double-sideband modulator is required to shift the spectrum down to the desired range for further frequency multiplication or transmission. This is shown in Figure 10.4 [1].

Figure 10.4 Indirect method of generating WFM.

Direct Method.

Here the carrier frequency is directly varied in accordance with the modulating signal. A common method used for generating direct FM is to vary the inductance L or capacitance C of a *voltage-controlled oscillator* (*VCO*). This is shown in Figure 10.5 [2].

Figure 10.5 Direct method of generating WFM.

The oscillator uses a high-Q resonant circuit. Variations in the inductance or capacitance of the oscillator will change its oscillating frequency. Assuming that the capacitance of the tuned circuit varies linearly with the modulating signal $m(t)$, we have

$$C = k\,m(t) + C_0 \qquad (10.17)$$

$$C = \Delta C + C_0 \qquad (10.18)$$

where

$$\Delta C = k\,m(t)$$

k is a constant and C_0 is the capacitance of the VCO when the input signal to the oscillator is zero. The instantaneous frequency is given by

$$f_i = \frac{1}{2\pi\sqrt{LC}} \qquad (10.19)$$

$$f_i = \frac{1}{2\pi\sqrt{LC_0}\sqrt{1+\dfrac{\Delta C}{C_0}}}$$

$$f_i = f_c \left(1 + \frac{\Delta C}{2C_0}\right)^{-1/2}$$

where the zero-input-signal resonance frequency is

$$f_c = \frac{1}{2\pi\sqrt{LC_0}} \tag{10.20}$$

For $\Delta C << C_0$, we can write

$$f_i \approx f_c\left(1 - \frac{\Delta C}{2C_0}\right)$$

$$f_i \approx f_c\left(1 - \frac{km(t)}{2C_0}\right) \tag{10.21}$$

$$f_i \approx f_c - \Delta f \tag{10.22}$$

where

$$\Delta f = \frac{km(t)}{2C_0} f_c = \frac{\Delta C}{2C_0} f_c \tag{10.23}$$

Although the change in capacitance may be small, the frequency deviation Δf may be quite large if the resonance frequency f_c is large. We can alternatively vary the inductance to achieve the same effect.

Advantage - Large frequency deviations are possible and thus less frequency multiplication is needed.

Disadvantage - The carrier frequency tends to drift and additional circuitry is required for frequency stabilisation.

To stabilise the carrier frequency, a phase-locked loop can be used. This is shown in Figure 10.6 [3].

Figure 10.6 Direct method of generating WFM with frequency stabilisation.

References

[1] H. P. Hsu, Analog and Digital Communications, McGraw-Hill, 1993.

[2] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw-Hill, 1990.

[3] L. W. Couch II, Digital and Analog Communication Systems, 5/e, Prentice Hall, 1997.

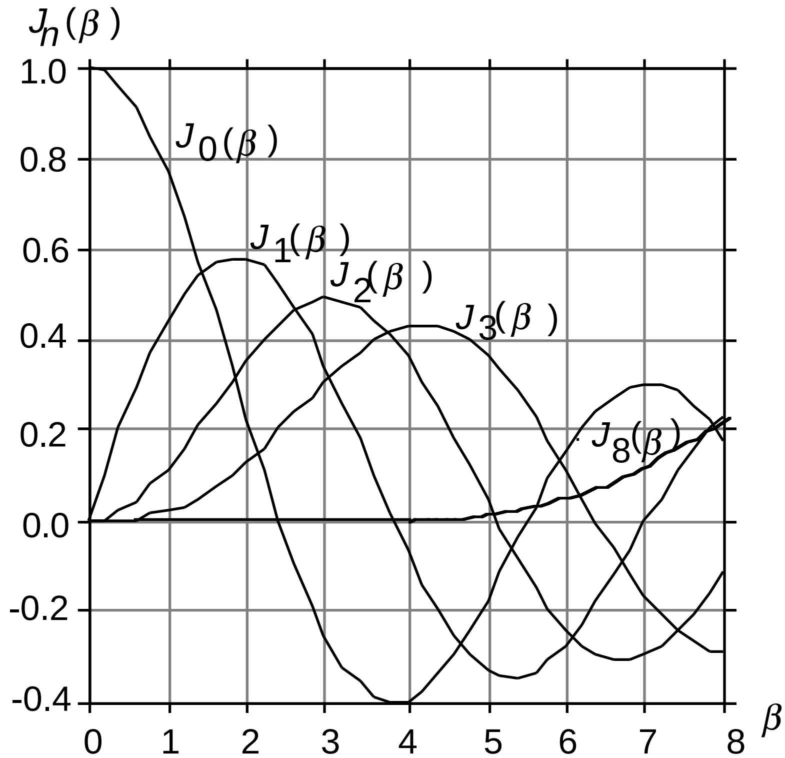


Figure 10.1 Bessel functions.

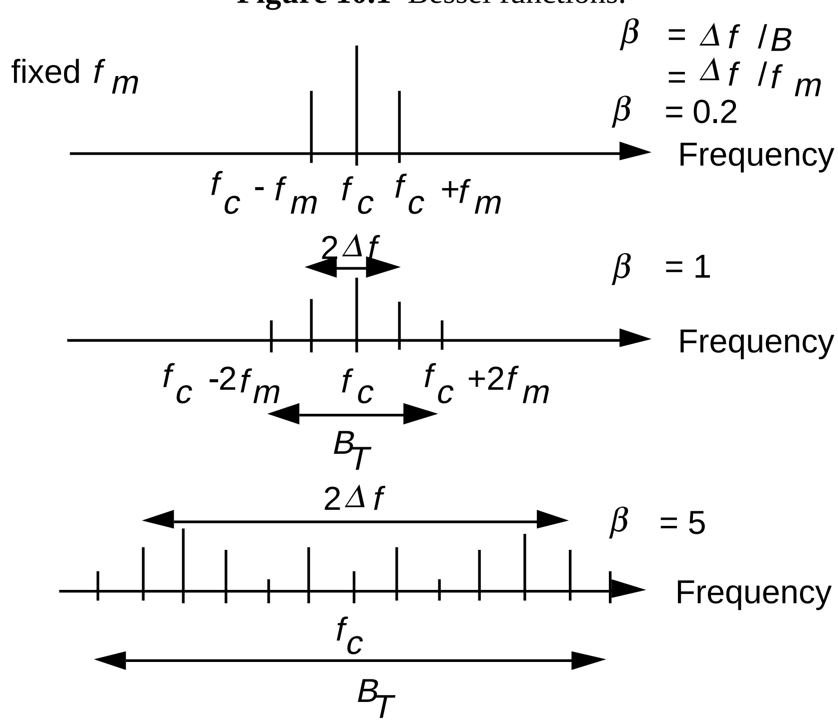


Figure 10.2 Amplitude spectra of FM signals with sinusoidal modulating signal and fixed f_m.

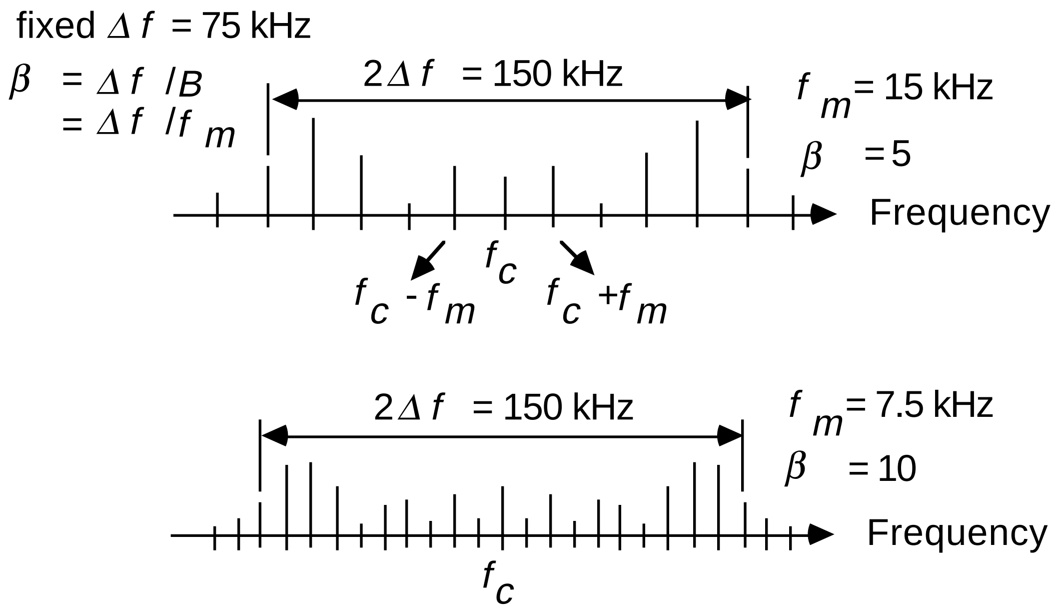


Figure 10.3 Amplitude spectra of FM signals with sinusoidal modulating signal and fixed peak frequency deviation Δf.

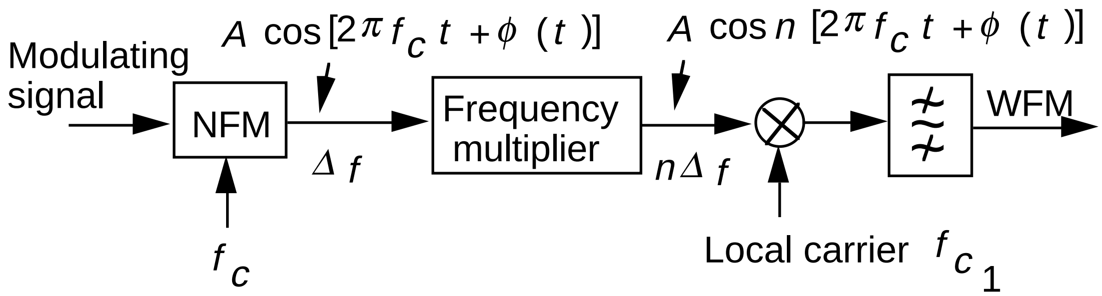
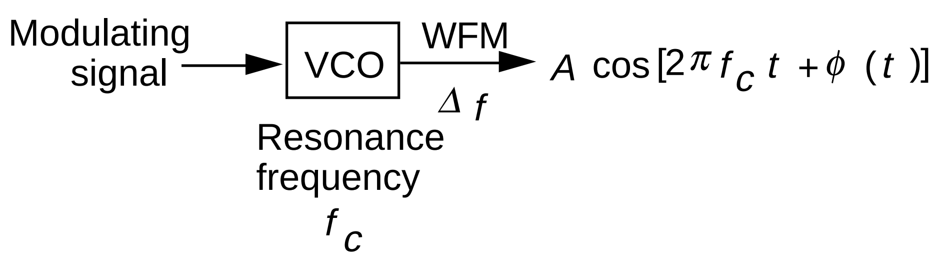


Figure 10.4 Indirect method of generating WFM.



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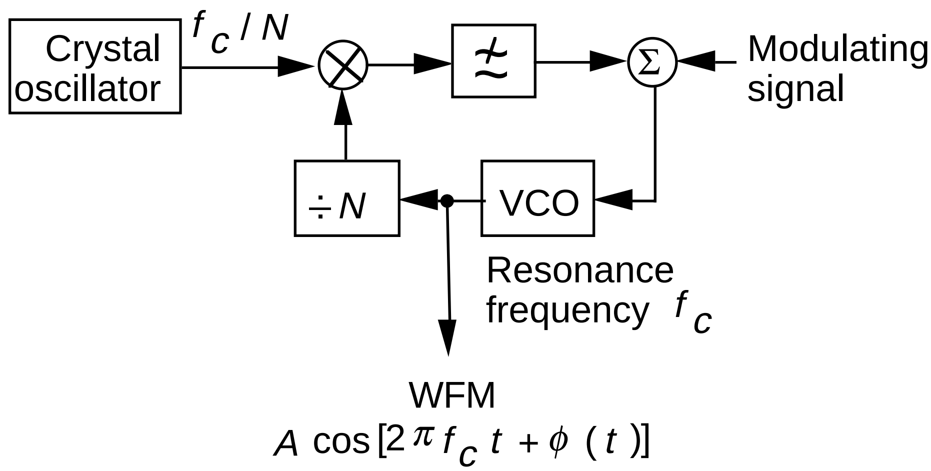

Figure 10.6 Direct method of generating WFM with frequency stabilisation.