# Tutorial Questions

**Week 1 (T1)** - 2.1a, 2.3, 2.10, and 2.31

2.1(a) Find the Fourier-series representations of each of the pulse trains in Fig. P2-1. Choose the time origin so that a cosine series is obtained in each case.

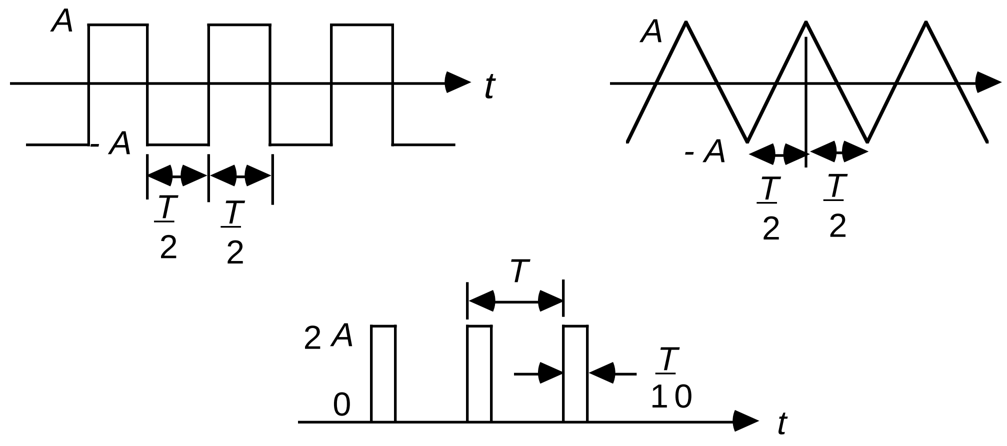


**Figure P2-1**

2.3 Find the complex Fourier series for the two pulse trains of Fig. P2-3. Plot and compare the two amplitude spectra. What is the significance of the term $e^{-j\omega_n t_0}$ ($\omega_n = 2\pi n/T$) in the expression for the complex Fourier coefficient for the rectangular pulses?

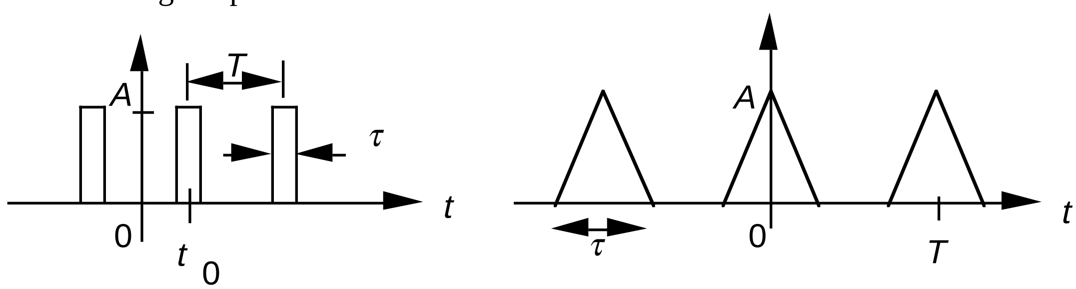


**Figure P2-3**

2.10 $T$ is the period of expansion in the Fourier-series representation for the function $f(t)$.

(a) Show that if $f(t) = f(t + T/2)$, the Fourier series will contain no odd harmonics.

(b) Show that if $f(t) = -f(t + T/2)$, the Fourier series will contain no even harmonics.

2.31 Find the frequency spectrum of the output voltage of Fig. P2-13. Leave the answer in complex form.

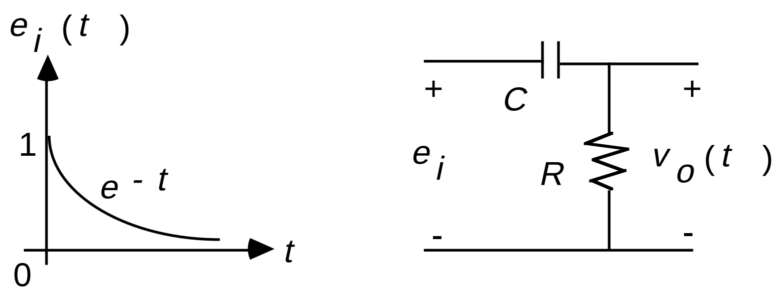


**Figure P2-13**

**Week 2 (T2)** - 2.35a, 2.41, and 2.47

2.35 A pulse of the form $e^{-at}u(t)$ with $u(t)$ the unit step function is applied to an idealised network with transfer characteristic $H(\omega) = Ae^{-jt_0\omega}$, $|\omega| \leq \omega_C$; $H(\omega) = 0$; $|\omega| > \omega_C$.

(a) Show that the output time response is given by

$$g(t) = \frac{A}{\pi}\int_0^{\omega_c}\left[\frac{a\cos\omega(t-t_0)}{a^2+\omega^2} + \frac{\omega\sin\omega(t-t_0)}{a^2+\omega^2}\right]d\omega$$

2.41 Suppose

$$H(\omega) = \frac{1}{a+j\omega} = |H(\omega)|e^{j\theta(\omega)}$$

(a) Sketch $|H(\omega)|$ and $\theta(\omega)$.

(b) Find and sketch the impulse response of a network with this transfer function.

(c) Show a simple $RC$ circuit that has a transfer function of this type. Compare the impulse response of the $RC$ circuit with the result of ($b$).

2.47 A time function $f(t)$ is multiplied by a periodic set of unit impulses, as shown in Fig. P2-47. The Fourier transform $F(\omega)$ is band-limited to (has no frequency components above) $B$ hertz as shown. Use frequency convolution to find the spectrum at the multiplier output. Sketch the three cases $1/T = 4B$, $1/T = 2B$, and $1/T = B$.

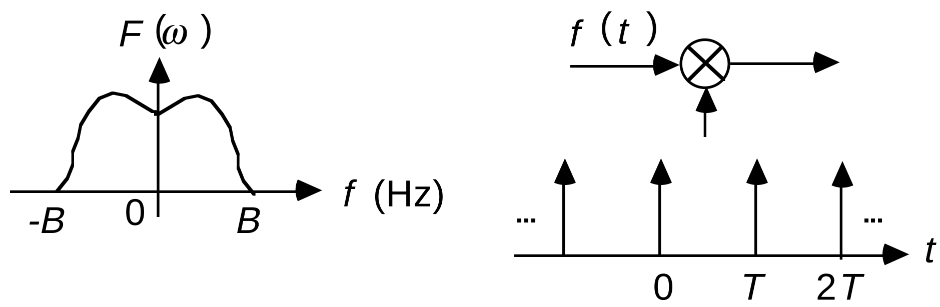


**Figure P2-47**

**Week 3 (T3)** - 4.37, 4.47, and 4.49

4.37. An AM signal has the form

$$s(t) = [1 + m\, f(t)] \cos \omega_0 t \qquad |m\, f(t)| \leq 1$$

The bandwidth of $f(t)$ is $B \ll f_0$. Consider the receiver shown in Fig. P4-37.

(a) Specify $f_x$, $B_x$, $f_1$, and $B_1$, if $f(t)$ is to be reproduced at the output.

(b) Show which of the three detectors shown could reproduce $f(t)$ at the output. Sketch carefully the corresponding filter characteristics. If the answer for device (2) is affirmative, specify what $f_2$ and $\theta$ should be.

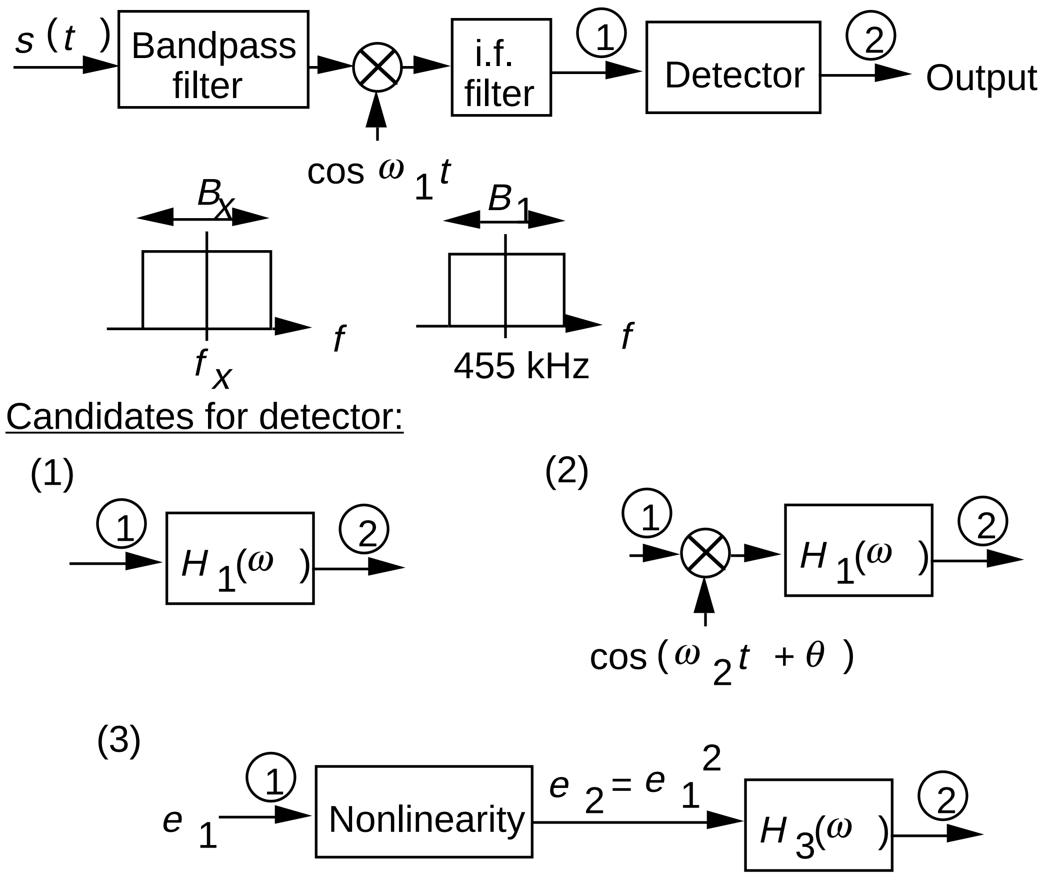


**Figure P4-37**

4.47. The radio receiver of Fig. P4-47 has the r-f amplifier and i-f amplifier characteristics shown. The mixer characteristic is given by $e_3 = e_2\,(\alpha_0 + \alpha_1 e_{01})$, with $\alpha_0$ and $\alpha_1$ constants, and $e_{01} = A \sin \omega_c{'}t$. The product-detector output is $e_5 = e_4\, e_{02}$, with $e_{02} = B \cos(\omega_c - \omega_c{'})t$. Find the signals at points 4 and 6 for each of the following signals at point 1:

(a) Normal AM, $(1 + \sin \omega_m t) \sin \omega_c t$.

(b) Double sideband, suppressed carrier, $\sin \omega_m t \sin \omega_c t$.

(c) Single sideband, $\cos(\omega_c - \omega_m)t$.

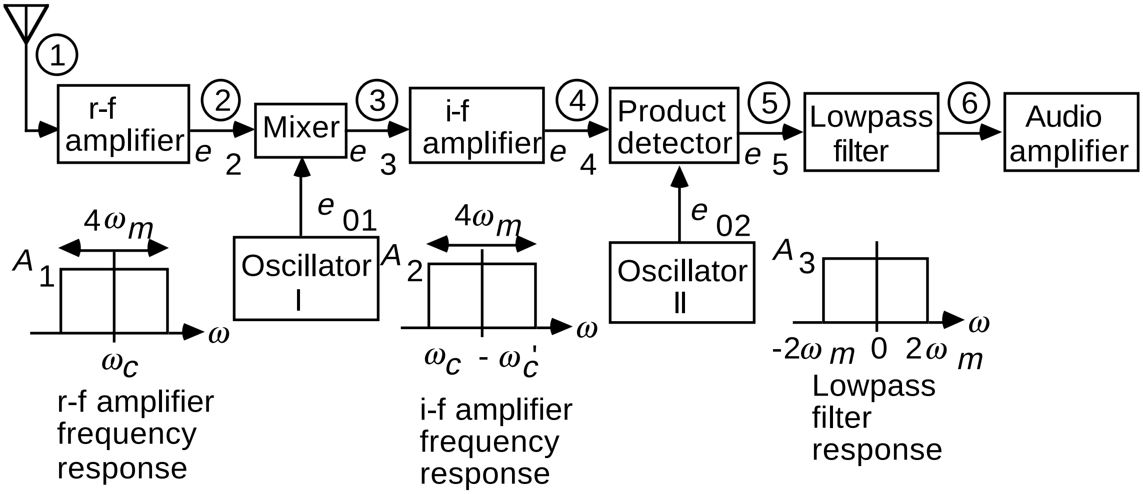


**Figure P4-47**

4.49. Consider the system shown in Fig. P4-49. Show that the output $g(t)$ is a SSB signal. Do this by assuming $F(\omega)$ as shown ( $B << f_c$) and carefully sketching the transforms at the output of each device. Preserve the distinction between the shaded and unshaded halves of $F(\omega)$. Is the lower or the upper half of a conventional DSB signal retained?

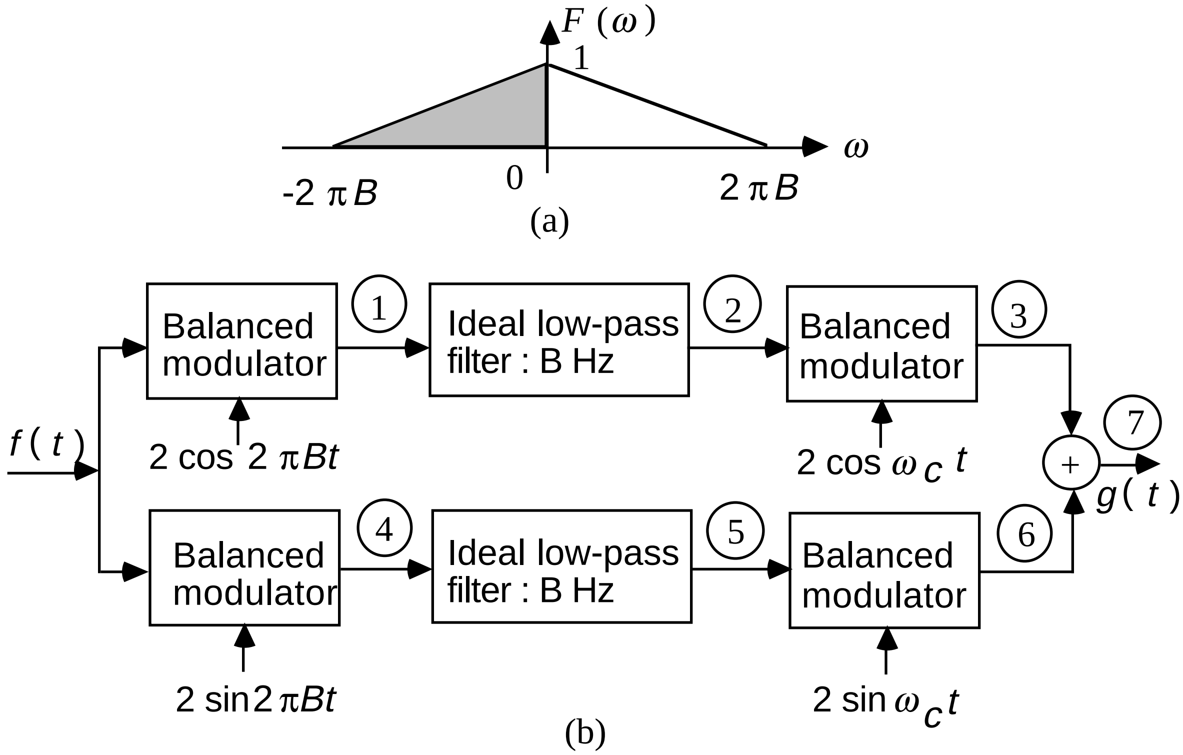


**Figure P4-49**

**Week 4 (T4)** - 4.55

4.55 An FM receiver similar to that of Fig. 4-64 is tuned to a carrier frequency of 100 MHz.

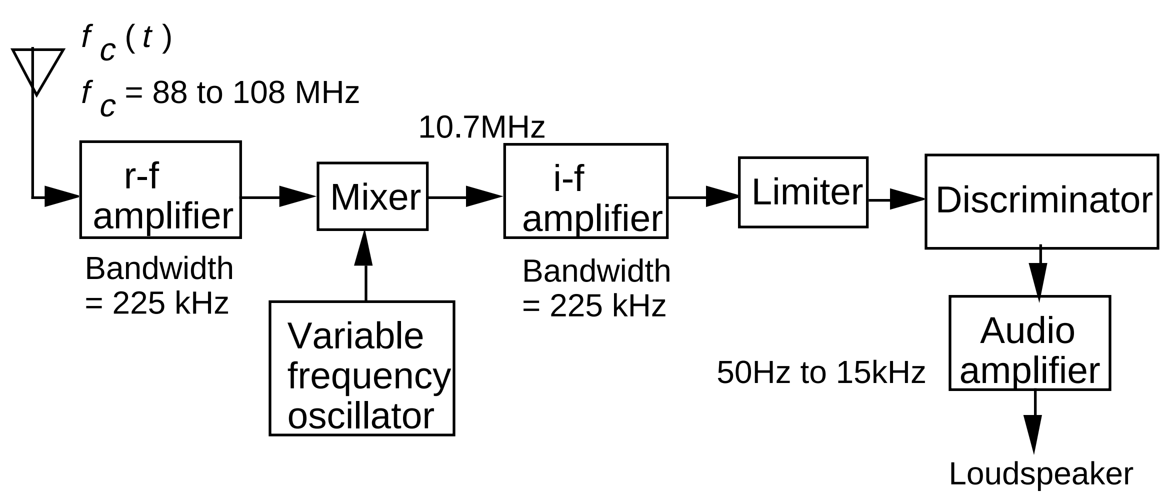


**Figure 4-64**

(a) A 10-kHz audio-frequency audio-signal modulates a 100-MHz carrier, producing a $\beta$ of 0.1. Find the bandwidths required of the r-f and i-f amplifiers and of the audio amplifier.

(b) Repeat (a) if $\beta = 5$.

(c) Two signals at 100 MHz are tuned in alternately. The carriers are of equal intensity. One is modulated with a 10-kHz signal and has $\beta = 5$; the other is modulated with a 2-kHz signal and has $\beta = 25$. Which requires the larger bandwidth? Explain. Compare the audio-amplifier outputs in the two cases.

(d) Two other signals are tuned in alternately. The carriers are again of equal intensity. One has a frequency deviation of 10 kHz with $\beta = 5$, the other a deviation of 2 kHz with $\beta = 25$. Which requires the larger bandwidth? Which gives the larger audio output?

**Week 5 (T5)** - 3.2 and 3.6

3.2 A function $f_1(t)$ is band-limited to 2,000 Hz, another function $f_2(t)$ to 4,000 Hz. Determine the maximum sampling interval if these two signals are to be time-multiplexed, using a single sampling rate.

3.6 Temperature measurements covering the range -40 to +40$^{o}$C, with $\pm$ 0.5$^{o}$C accuracy, are taken as 1-s intervals. They are converted to binary format for PCM transmission. What is the bit rate required?

**Week 6 (T6)** - 3.5, 3.8, and 3.11

3.5 Five signal channels are sampled at the same rate and time-multiplexed. The multiplexed signals is then passed through a low-pass filter. Three of the channels handle signal covering the frequency range 300 to 3,000 Hz. The other two carry signals covering the range 50 Hz to 10 kHz. Included is a synchronisation signal.

(a) What is the minimum sampling rate?

(b) For this sampling rate what is the minimum bandwidth of the low-pass filter?

3.8 Consider the PCM system shown in Fig. P3-8, which time-multiplexes 10 signal channels.

(a) What is the minimum sampling rate?

(b) 30,000 samples/s are taken.

(1) What is the bit rate (bits/s) at the PCM output?

(2) What are the *minimum* bandwidths required at points 1, 2, 3?

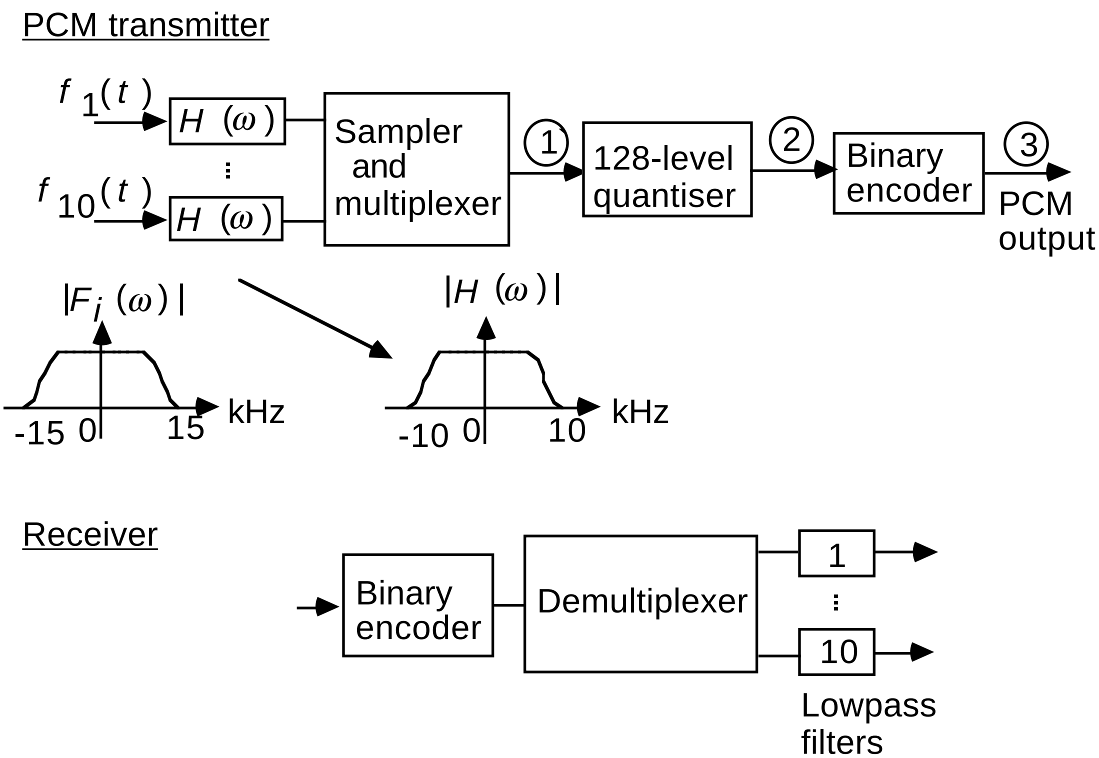


**Figure P3-8**

3.11 A single information channel carries voice frequencies in the range of 50 to 3,300 Hz. The channel is sampled at 8 kHz rate, and the resulting pulses are transmitted over either a PAM system or a PCM system.

(a) Calculate the minimum bandwidth of the PAM system.

(b) In the PCM system, the sampled pulses are quantised into eight levels and transmitted as binary digits. Find the transmission bandwidth of the PCM system, and compare with that of (a).

(c) Repeat (b) if 128 quantising levels are used. Compare the rms quantisation noise in the two cases if the peak-to-peak voltage swing at the quantiser is 2V.

**Week 7 (T7)** - 3.14, 3.24, 2.8, and 3.9

3.14 A signal voltage in the frequency range 100 to 4,000 Hz is limited to a peak-to-peak swing of 3 V. It is sampled at a uniform rate of 8 kHz, and the samples are quantised to 64 evenly spaced levels. Calculate and compare the bandwidths and ratios of peak signal to rms quantisation noise if the quantised samples are transmitted either as binary digits or as four-level pulses.

3.24 12 voice signals covering the range 20 Hz - 20 kHz are low-pass filtered to 7 kHz, and are then multiplexed into one binary stream as shown in Fig. P3-24a.

(a) What is the minimum sampling rate?

(b) The signals are each sampled at a rate of 16 ksamples/s. 256 levels of quantisation are used. Framing bits are ignored. Show the output rate is 1.536 Mbits/s.

(c) The roll-off factor $r$ = 0.25. Find the bandwidth required to transmit the baseband pulses.

(d) The receiver is diagrammed in Fig. P3-24b.
(1) Specify the type of filter and its bandwidth.
(2) Provide two reasons why $\hat{f}_i(t)$ differs from $f_i(t)$, i = 1, ..., 12.

(e) Frames are 125 $\mu$s long. Eight framing bits are added per frame. Repeat (c).

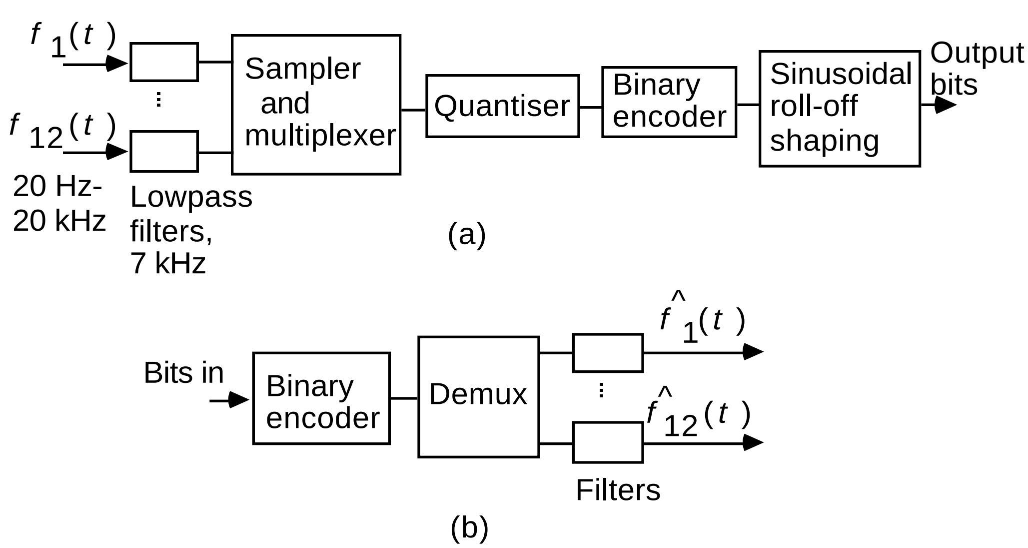


**Figure P3-24**

2.8 Find the complex Fourier series for the two periodic functions of Fig. P2-8.

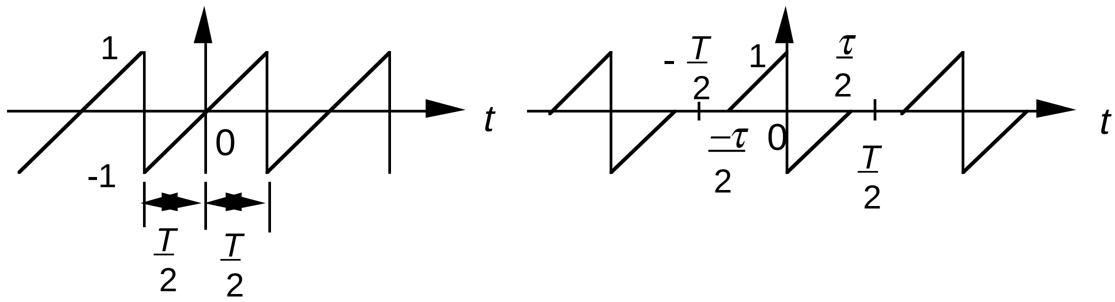


**Figure P2-8**

3.9 Ten 10-kHz signal channels are sampled and multiplexed at a rate of 25,000 samples/s per channel. Each sample is then coded into six binary digits.

(a) Find the PCM output rate, in bits/s. Estimate the bandwidth needed to transmit the PCM stream.

(b) Using the same number of quantisation levels as above, each sample is now transmitted as a sequence of four-level pulses. What is the output rate, in bits/s? Estimate the transmission bandwidth required.

(c) Repeat (b) for the case in which each quantised sample is transmitted as a multilevel pulse without any further coding.

**Week 8 (T8)** - 4.5, 4.9, 4.11, and 4.18

4.5 A telephone channel allows signal transmission in the range 600 to 3,000 Hz. The carrier frequency is taken to be 1,800 Hz.

(a) Show that 2,400 bit/s, 4PSK transmission with raised cosine shaping is possible. Show that the 6 dB bandwidth about the carrier is 1,200 Hz.

(b) 4,800 bits/s are to be transmitted over the same channel. Show that 8PSK, with 50% sinusoidal roll-off, will accommodate the desired date rate. Show that the 6 dB bandwidth about the carrier is now 1,600 Hz.

4.9 A single voice channel is to be transmitted via PCM techniques using satellite communications. 8,000 samples/s are taken. and 7-bit quantisation (128 levels) is used. 32 synchronisation bits are inserted into the binary stream for every 224 data bits transmitted. The resultant binary stream is then transmitted using sinusoidal roll-off shaping for each binary pulse. The roll-off factor is 20%.

(a) What is the PCM bit rate in bits/s?
(b) What is the baseband (PCM signal) bandwidth?
(c) PSK is used for transmission. What is the transmission bandwidth required?
(d) 4PSK is used instead: successive *pairs* of bits are used to phase-modulate a carrier. What is the transmission bandwidth required in this case?
(e) Repeat (c) if OOK transmission is used. Repeat if FSK is used with the frequency deviation chosen as 38 kHz. What is the modulation index $\beta$ in this case? What is the frequency spacing of the two cases?

4.11 Ten 32-kbit/s delta-modulated voice channels and four 64-kbits/s computer outputs are multiplexed using TDM (time-division multiplexing). TDM frames are 125 $\mu$s long. 8 control bits per frame are added.

(a) Show that the bit rate of the TDM output is 640 kbits/s.
(b) The TDM output is to be transmitted over a radio channel centred at 10 MHz. A transmission bandwidth of 240 kHz at this carrier frequency is available. Specify a modulation technique that may be used if sinusoidal roll-off shaping with $r = 0.5$ is used. Show all work.
(c) Repeat (b), using a different modulation technique, with the roll-off factor reduced to $r = 0.1$.

4.18 A transmission channel of 1 MHz bandwidth, centred at 100 MHz, is available for signal transmission.

(a) The PCM system and 240 kbits/s data source of Fig. P4-18 are time-multiplexed together, and the output then fed into a PSK modulator, as shown (25% roll-off shaping is used). Find the maximum number of quantisation levels in the PCM system that may be used. (Neglect framing, signalling, and stuffing bits in this example.)

(b) The PCM system in (a) is required to have 256-level quantisation. *Two* 240-kbits/s data sources are to be time-multiplexed with the PCM output. The same transmission channel must again be used. Indicate how the modulator might be modified to accomplish this.

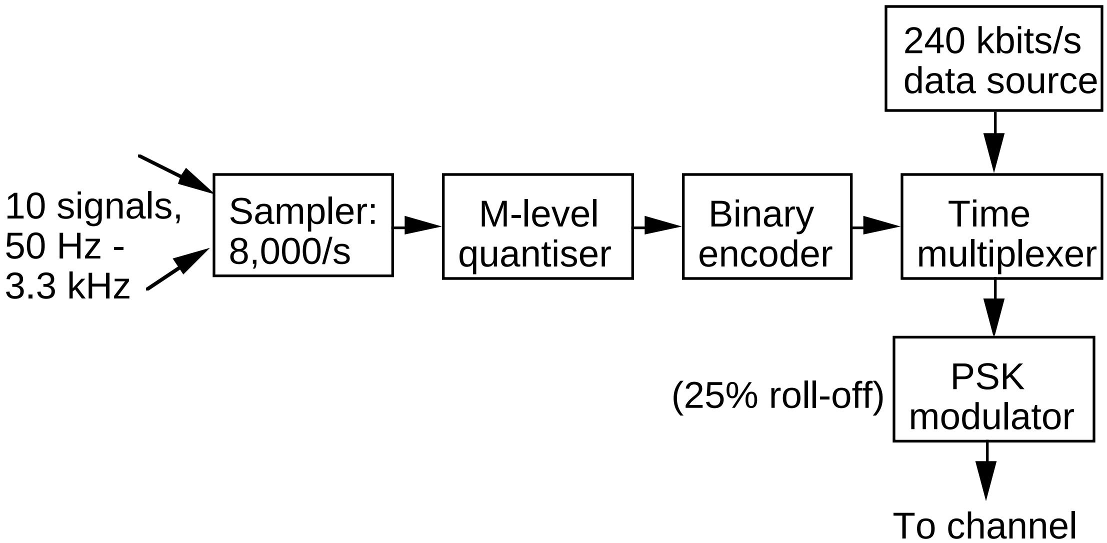


**Figure P4-18**

**Week 9 (T9)** - 6.9 and 6.20

6.9 A binary transmission system transmits 50,000 digits per second. Fluctuation noise is added to the signal in the process of transmission so that, at the decoder, where the digits are converted back to a desired output form, the signal pulses are 1 V in amplitude, with the rms noise voltage 0.2 V. What is the average time between mistakes of this system? How is this average time changed if the signal pulses are doubled in amplitude? *Note*: Assume that 1's and 0's are equally likely to be transmitted.

6.20 Band-limited white noise $n(t)$ has spectral density $G_n(f) = 10^{-6}\ V^2/Hz$, over the frequency range -100 to +100 kHz.

(a) Show that the rms value of the noise is approximately 0.45 V.

(b) Find $R_n(\tau)$. At what spacings are $n(t)$ and $n(t+\tau)$ uncorrelated?

(c) $n(t)$ is assumed Gaussian. What is the probability at any time $t$ that $n(t)$ will exceed 0.45 V? 0.9 V?

(d) The noise, again assumed Gaussian, is added to polar signals of amplitude $\pm A$. What is the probability of error if the binary signals are equally likely, the decision level is taken as 0, and $A$ = 0.9 V? Repeat for $A$ = 1.8 V and 4.5 V.

**Week 10 (T10)** -6.33

6.33 An on-off binary sequence $s(t)$ has white Gaussian noise $n(t)$ added to it, as shown in Fig. P6-33. 1,000 bits/s are transmitted. A typical binary 1 has the shape shown in the figure (50% roll-off sinusoidal shaping is used). $H(\omega)$ may be taken to have zero phase. The decision circuit shown outputs a 1 if $v > A/2$ and a 0 if $v < A/2$, with $v$ the value of the voltage at the output of $H(\omega)$, taken once every millisecond at the expected maximum $A$ of the signal component of $v(t)$. Find the probability of error of this system, if 1's and 0's are equally likely to be transmitted.

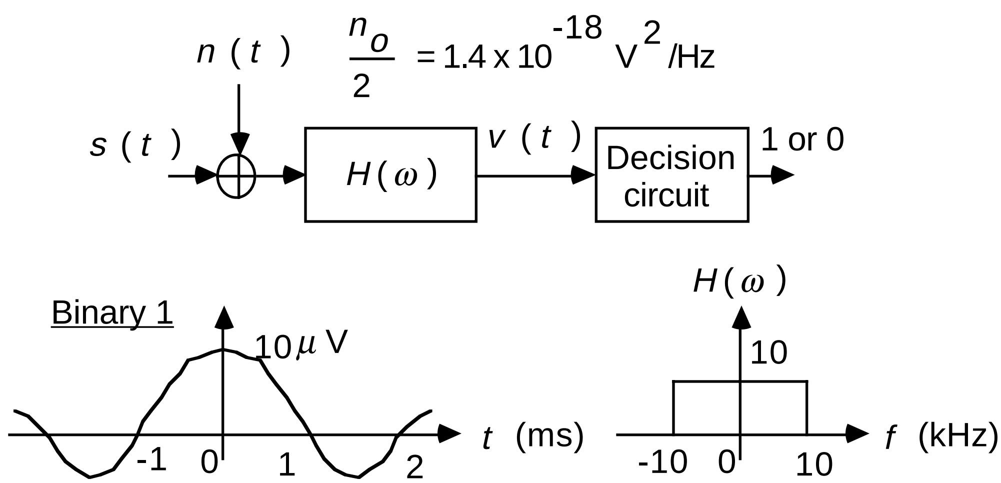


**Figure P6-33**

**Week 11** - Nil

**Week 12** - Nil

**Week 13** - Nil

**Reference**

[1] M. Schwartz, Information Transmission, Modulation, and Noise, 4/e, McGraw-Hill, 1990.