INFO240 Signal Analysis and Processing - Assignment 4

Submit to the INFO240 assignment box opposite E6A Room 247

For this assignment the Fourier transform operator $\mathcal{F}$ is defined as

$$F(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \mathcal{F}^{-1}[F(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

which establishes $f(t)$ and $F(\omega)$ as time-domain and frequency-domain views of a signal.

1. Use the *duality* and *time reversal* properties of the Fourier transform operator to show that

   (a) o If $\Psi(\omega) = \mathcal{F}[\Phi(t)]$, then $\mathcal{F}[\Psi(t)] = 2\pi\Phi(-\omega)$,

   (b) o and if $\Phi(t) = \mathcal{F}^{-1}[\Psi(\omega)]$, then $\mathcal{F}^{-1}[\Phi(\omega)] = \Psi(-t)/2\pi$.

2. Consider a raised-cosine pulse, $c(t)$ defined by: $c(t) = \begin{cases} 1+\cos\pi t & -1 < t < 1 \\ 0 & otherwise \end{cases}$

   (a) o Calculate the Fourier transform $C(\omega) = \mathcal{F}[c(t)]$.

   (b) o Calculate the Fourier transform of a periodic cosine signal $S(\omega) = \mathcal{F}[1 + \cos \pi t]$.

   (c) o Calculate the Fourier transform of a rectangular pulse

   $$W(\omega) = \mathcal{F}[w(t) = \begin{cases} 1 & -1 < t < 1 \\ 0 & otherwise \end{cases}]$$

   (d) o Confirm that $C(\omega) = \frac{1}{2\pi} S(\omega) * W(\omega)$.

3. A short burst of computer data is to be transmitted over a local area network. The bust of data is the sequence {0, 1, 0, 1, 0, 1} represented by a three cycle burst of a 5MHz sinusoid:

   $$d(t) = \begin{cases} \sin 10^7 \pi t & for\ -1.5\times10^{-7} < t < 1.5\times10^{-7} \\ 0 & otherwise \end{cases}$$

(a) o Sketch the data burst, showing time values and label the zero and one data bits.

(b) oo Express the data bust as the product of a continuous sinusoid and a rectangular pulse. Then, determine the Fourier transform of the data burst by convolution of the transforms of the continuous sinusoid and the rectangular pulse.

(c) ooo If most of the power of a sin x/x signal is contained in its main *lobe*, estimate the range of frequencies that must be transmitted in order to send the burst of data.
Why is it necessary for a local area network to have a bandwidth that is wider than half the maximum data rate?

A total of ten marks, one for each 'o', will be awarded for this assignment.