MACQUARIE
UNIVERSITY ~ SYDNEY

END-OF-YEAR EXAMINATIONS 2004

Unit: **INFO240 Signal Analysis and Processing (D3) - Second Paper**

Date: Monday, 29 November 2004, 1:50 p.m.

Time allowed: One hour and 30 minutes plus 10 minutes reading time.

Total Number of Questions: TWO (2)

Instructions: Both questions should be attempted.
There is a total of 30 marks for the paper.
The questions are of equal value (15 marks each).

Materials Permitted: The use of non-programmable electronic calculators is permitted. No dictionaries are permitted.

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1. Signals and Systems

(a) (2 marks)
Consider the following two discrete-time signals:

$$q[k] = \begin{cases} 0 & k < 0 \\ k+1 & 0 \le k \le 3 \\ 0 & k > 3 \end{cases} \quad \text{and} \quad r[k] = \begin{cases} 0 & k < 0 \\ 1 & 0 \le k \le 3 \\ 0 & k > 3 \end{cases}$$

Sketch the following two signals:

$$f_1[k] = 0.5\, q[k + 2] + r[-k],$$

and

$$f_2[k] = 0.5\, q[- k - 2] - r[-k + 1].$$

(b) (3 marks)

(i) State the necessary property that a continuous-time signal $f\{t)$ must have for it to be periodic with a period P.

(ii) State the necessary property that a discrete-time signal $y[k]$ must have for it to be periodic with a period N.

(iii) Determine the fundamental periods of $f(t) = 5 \sin(\pi t + \frac{\pi}{2})$ and $y[k] = \sin \pi k$.

(c) (3 + 3 = 6 marks)
Consider an IIR system, $u[k] \rightarrow y[k]$, $k \geq 0$, that has a zero-state impulse response of

$$h[k] = \frac{1}{2^k}.$$

Show that the difference equation of the system is

$$y[k] - 0.5\, y[k - 1] = u[k].$$

Draw a diagram to show how this system might be implemented using blocks that perform multiplication, addition and delay functions.

(d) (4 marks)
Find the convolution of the two continuous-time functions $f(t)$ and $g(t)$ below:

$$f(t) = \begin{cases} e^{at} & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad g(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \le t < 1. \\ 0 & t \geq 1 \end{cases}$$

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2. Signal Analysis

(a) (4 marks)
Consider the following two signals

$f_1(t) = \sin 5t \times \cos t$ $\qquad$ $f_2(t) = \sin 4t + 2 \cos 12\pi t$.

(i) Only one of these signals is periodic; determine which one it is and justify your claim.

(ii) Determine the fundamental frequency of this periodic signal and hence a suitable value for its fundamental angular frequency ω_0.

If the Fourier series of a periodic signal $f(t)$ is witten as

$$f(t) = \sum_{m=-\infty}^{\infty} c_m e^{jm\omega_0 t}, \text{ then}$$

(iii) Determine the Fourier sequence $\{c_m\}$ of the periodic signal found in (i).

(iv) Plot the discrete frequency spectrum of this periodic signal.

(b) (3 marks)
The Fourier transform operator $\mathcal{F}$ is definded as

$$F(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$
$$f(t) = \mathcal{F}^{-1}[F(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

which established $f(t)$ and $F(\omega)$ as time-domain and frequency-domain views of a signal. Show that if $F(\omega) = \mathcal{F}[f(t)]$ then

$$\mathcal{F}[f(-t)] = F(-\omega).$$

[Question 2 continues on the next page.]

[Question 2 continued from the previous page.]

(c) (8 marks)

A time function $f(t)$ is multiplied by a periodic set of unit impulses, as shown in Figure 2. The Fourier transform $F(\omega)$ is band-limited to (has no frequency components above) B hertz as shown. Use frequency convolution to find the spectrum at the multiplier output. Sketch the three cases $1/T = 4B$, $1/T = 2B$, and $1/T = B$.

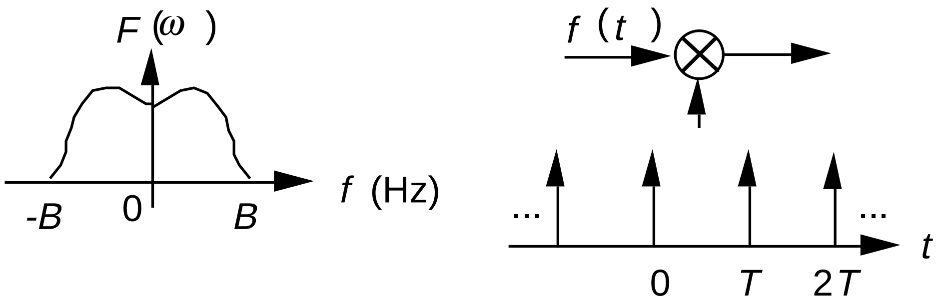

Figure 2.
