MACQUARIE
UNIVERSITY ~ SYDNEY

MID-YEAR EXAMINATIONS 2005

Unit:	**INFO240 Signal Analysis and Processing (D3) - First Paper**
Date:	Tuesday, 28 June 2005, 1:50 p.m.
Time allowed:	One hour and 30 minutes plus 10 minutes reading time.
Total Number of Questions:	TWO (2)
Instructions:	Both questions should be attempted. There is a total of 30 marks for the paper. The questions are of equal value (15 marks each).
Materials Permitted:	The use of non-programmable electronic calculators is permitted. No dictionaries are permitted.

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1. Signals and Systems

Write concise but informative notes, in the context of signal and system analysis, of the following four terms.

(a) **(1 mark)** Memory system

(b) **(1 mark)** Causal system

(c) **(1 mark)** Linear system

(d) **(1 mark)** Time-invariant system

(e) **(2 marks)**

An arbitrary signal $x(t)$ can always be expressed as a sum of even and odd signals as $x(t) = x_e(t) + x_o(t)$, where $x_e(t)$ is called the even part of $x(t)$ and $x_o(t)$ is called the odd part of $x(t)$. Show that $x_e(t) = 0.5[x(t) + x(-t)]$ is an even signal and $x_o(t) = 0.5[x(t) - x(-t)]$ is an odd signal.

(f) **(3 marks)**

(i) State the necessary property that a discrete-time signal $y[k]$ must have for it to be periodic with a period N.

(ii) Let $x[k]$ be the sum of two periodic signals $x_1[k]$ and $x_2[k]$, with periods N_1 and N_2, respectively. Let p and q be two integers such that $pN_1 = qN_2 = N$. Show that the discrete-time signal $x[k]$ is also periodic with period N.

(g) **(6 marks)**

Determine the continuous-time convolution $y(t) = f(t) * g(t)$ analytically where

$$f(t) = A\, e^{-t}, \qquad 0 \le t < \infty$$

and

$$\begin{cases} g(t) = t/T, & 0 \le t < T, \\ \quad\;\; = 0, & \textit{otherwise} \end{cases}$$

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2. Signal Analysis

(a) (9 marks)

(i) Determine which one of the following signals can be represented by a Fourier series and justify your claim. **(2 marks)**

$$f_1(t) = \cos 6t + \sin 8t + e^{j2t}$$
$$f_2(t) = \cos t + \sin \pi t$$

(ii) Determine the fundamental frequency of the periodic signal found in (i) and hence a suitable value for its fundamental angular frequency ω_0. **(1 mark)**

If the Fourier series of a periodic signal $f(t)$ is written as

$$f(t) = \sum_{m=-\infty}^{\infty} c_m e^{jm\omega_0 t},$$

and the average power (energy per unit time) in a signal with period $P = 2\pi/\omega_0$ is defined as

$$P_{av} = \frac{1}{P} \int_{<P>} f(t)\, f^*(t)\, dt$$

(iii) Show that $P_{av} = \sum_{m=-\infty}^{\infty} |c_m|^2$. **(3 marks)**

(iv) Determine the Fourier sequence $\{c_m\}$ and the average power of the periodic signal found in (i). **(2 marks)**

(v) Plot the discrete frequency spectrum of the periodic signal. **(1 mark)**

(b) (3 marks)

The Fourier transform operator $\mathcal{F}$ is defined as

$$F(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$
$$f(t) = \mathcal{F}^{-1}[F(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

which establishes $f(t)$ and $F(\omega)$ as time-domain and frequency-domain views of a signal. Show that $f(\omega) = \frac{1}{2\pi} \mathcal{F}[F(-t)]$.

[Question 2 continues on the next page]

[Question 2 continued from the previous page]

(c) (3 marks)

Compute the DFT of the following N-point sequences:

(i) $x[n] = \begin{cases} 1, & n = n_0, \quad 0 < n_0 < N-1 \\ 0, & \text{otherwise} \end{cases}$

(ii) $x[n] = (-1)^n$

(iii) $x[n] = \begin{cases} 1, & n \quad \text{even} \\ 0, & \text{otherwise} \end{cases}$
