MACQUARIE
UNIVERSITY ~ SYDNEY

END-OF-YEAR EXAMINATIONS 2006

Unit: **INFO240 Signal Analysis and Processing (D3) - Second Paper**

Date: Monday, 27 November 2006, 9:20 a.m.

Time allowed: One hour and 30 minutes plus 10 minutes reading time.

Total Number of Questions: TWO (2)

Instructions: Both questions should be attempted.
There is a total of 30 marks for the paper.
The questions are of equal value (15 marks each).

Materials Permitted: The use of non-programmable electronic calculators is permitted. No dictionaries are permitted.

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1. Signals and Systems

(a) (4 marks)

Determine whether the system described by

$$y(t) = x(t) \times \sin 2t$$

is

(i) memoryless;
(ii) causal;
(iii) time-invariant;
(vi) linear.

(b) (1 mark)

State the necessary condition for a discrete-time signal $x[k]$ to be periodic with a period N when $x[k]$ is constructed by sampling the continuous-time signal $x(t) = \sin \omega t$ with a sampling period T.

(c) (4 marks)

A continuous-time signal $x(t) = \cos 2\pi t$ is sampled every T seconds, resulting in the discrete-time signal $x[k] = x(kT)$. Determine whether the sampled signal is periodic for $T = 1\ s$ and $T = 0.13\ s$.

For those sampled signals that are periodic:

(i) find the number of periods of $x(t)$ in one period of $x[k]$
(ii) find the number of samples in one period of $x[k]$.

(d) (6 marks)

Determine the continuous-time convolution $y(t) = f(t) * g(t)$ analytically where

$$\begin{cases} f(t) = 1, & -a \le t \le a, \\ \quad = 0, & otherwise \end{cases}$$

and

$$\begin{cases} g(t) = 1, & -a \le t \le a, \\ \quad = 0, & otherwise. \end{cases}$$

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2. Signal Analysis

(a) (4 marks)

Determine whether the following signal can be represented by a Fourier series and justify your claim.

$f(t) = \cos 3t + \sin 5t$

(i) Determine the fundamental frequency of this periodic signal and hence a suitable value for its fundamental angular frequency ω_0.

(ii) Determine the Fourier sequence $\{c_m\}$ and the average power of the periodic signal found in (i).

(iii) Plot the discrete frequency spectrum of this periodic signal.

(b) (5 marks)

The Fourier transform operator $\mathcal{F}$ is definded as

$$F(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \mathcal{F}^{-1}[F(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

which establishes $f(t)$ and $F(\omega)$ as time-domain and frequency-domain views of a signal. Show that $F(t) = 2\pi\, \mathcal{F}^{-1}[f(-\omega)]$.

(c) (6 marks)

The Discrete Fourier Transform pair is given by

$$f[k] = \frac{1}{N} \sum_{m=0}^{N-1} F[m] e^{j2\pi mk/N}$$

$$F[m] = \sum_{k=0}^{N-1} f[k] e^{-j2\pi mk/N}$$

Compute the DFT of the following N-point sequence ($N = 4$):

$f[n] = n + 1,\ 0 \le n \le 3.$
