

15 Signal Manipulation

Signals can be manipulated by a *transform* of the independent variable of the function describing the signal

$$g(t) = f(F(t))$$

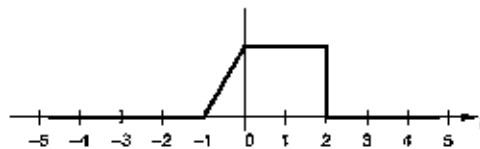
The independent variable remains t . The signal $g(t)$ is described by a function that is a *function of a function*.

15.1 Shifting

Shifting is a simple manipulation that transforms the independent variable by adding a constant.

Example

$f(t)$



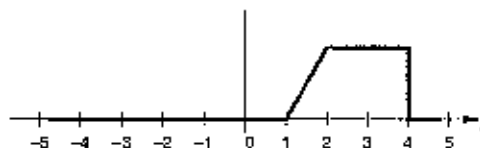
left shift

$f(t + 1)$



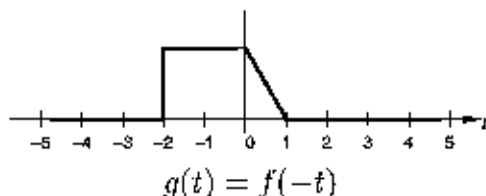
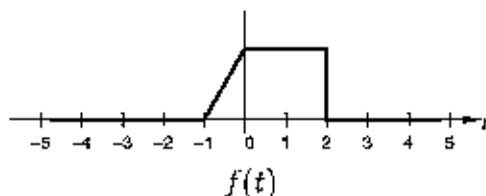
right shift

$f(t - 2)$



15.2 Flipping

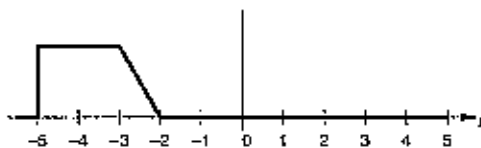
Transforming t by negating flips the signal with respect to $t = 0$.



15.3 Flipping and shifting

Flip then shift

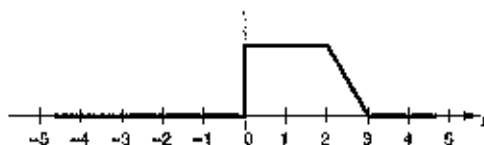
$$f(-(t+3)) = f(-t-3)$$



Starting with $f(t)$, the flipped function is $f(-t)$. The argument of the flipped function is t , NOT $-t$.

Shift then flip

$$f((-t)+2) = f(-t+2)$$



Starting with $f(t)$, the shifted function is $f(t+2)$. The argument of the shifted function is t , NOT $(t+2)$.

15.4 Multiplication

Definition:

Given two signals:

$$f(t) \text{ and } h(t) \quad \forall t \in (-\infty, \infty),$$

the product of these signals is

$$g(t) = h(t)f(t).$$

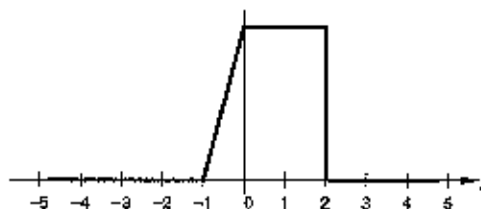
In communications technology, $h(t)$ is said to *modulate* the signal $f(t)$.

Special Cases

Amplification

$h(t)$ is constant.

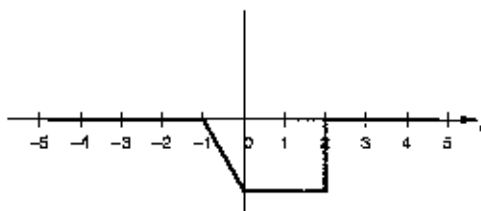
$$g(t) = 2f(t)$$



Inversion

$$h(t) = -1.$$

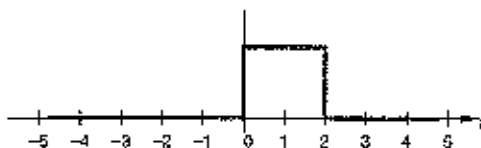
$$g(t) = -f(t)$$



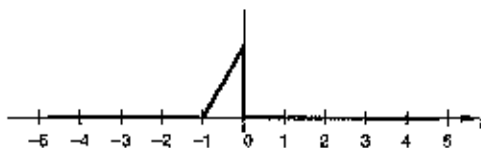
Truncation

$h(t) = q(t)$, which is the unit step function: $\begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$.

$$g(t) = q(t)f(t)$$



$$g(t) = q(-t)f(t)$$



15.5 Positive and negative-time signals

A signal $f(t)$ can be considered as the sum of a *negative-time* part $f_-(t)$ and a *positive-time* part $f_+(t)$.

$$f_+(t) = q(t)f(t) = \begin{cases} f(t) & t > 0 \\ 0 & t < 0 \end{cases}$$

$$f_-(t) = q(-t)f(t) = \begin{cases} 0 & t > 0 \\ f(t) & t < 0 \end{cases}$$

If $g(t) = 0 \quad \forall t < 0$ then $g(t)$ is a *positive-time* signal.

If $g(t) = 0 \quad \forall t > 0$ then $g(t)$ is a *negative-time* signal.

Otherwise $g(t)$ is *two-sided*.

Most signals are positive-time if $t = 0$ is defined as the instant when the signal starts.

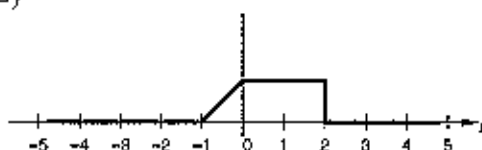
15.6 Signal Construction

Complicated signals can be constructed by summing basic signals that have been manipulated as above.

Example

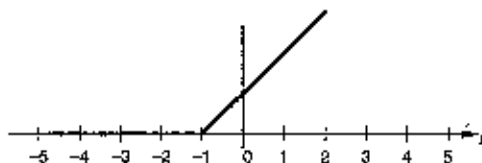
$f(t)$ used above is the sum of ramp and step functions:

$$f(t) = r(t + 1) - r(t) - q(t - 2)$$



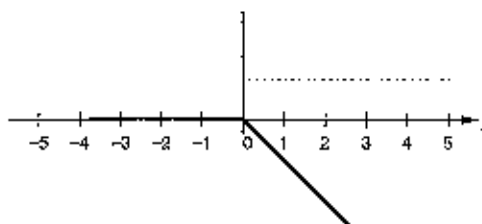
Shifted ramp

$$r(t + 1)$$



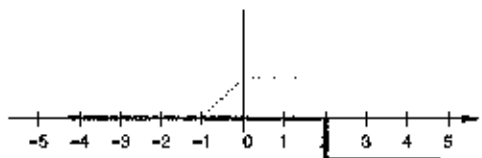
Inverted ramp

$$-r(t)$$



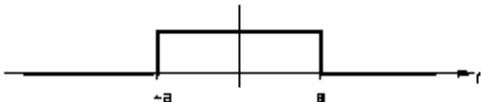
Inverted, shifted step

$$-q(t - 2)$$

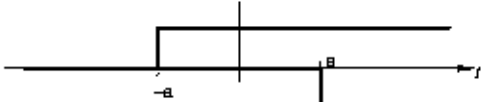


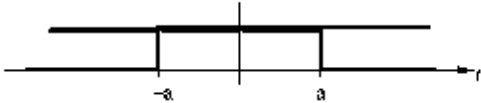
Rectangular pulse

Definition:

$$P_a(t) = \begin{cases} 1 & -a < t < a \\ 0 & t < -a \text{ or } t > a \end{cases}$$


The rectangular pulse can be constructed from step functions:

$$P_a(t) = q(t+a) - q(t-a)$$


$$P_a(t) = q(t+a) \times q(-t+a)$$


15.7 Windowing

Real signals are observed in finite time intervals — starting when the observation begins, and stopping when the observation ends. A portion of the signal is observed in a *window*.

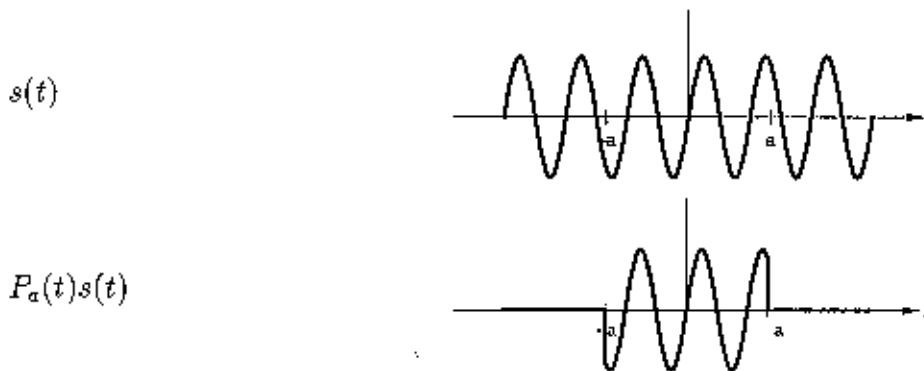
Windowing

A window function $W(t)$ is one that is zero for all time outside a certain time interval.

A signal is windowed when multiplied by a *window* function:

$$s_w(t) = W(t)s(t)$$

The signal can exist, or be defined, for all time. The portion that is observed is constructed by windowing it. The simplest window is a rectangular pulse:



A shifted rectangular pulse gives a observation window starting at $t = 0$:

